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Approximation techniques of stress concentration factor of double notched plate

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Abstract

The purpose of this study is to understand the approximation method of the stress concentration at the root of double notches. A series of models has been developed, and these stress distributions were computed by 2-dimensional body force method. The calculation models were a finite center notched, and side edge notched plate under uniform tensile stress. Three different stress concentration factors; i.e., $K_{\mathrm{t,exact}}$ for direct calculation, $K_{\mathrm{t,double}}$ for approximation by the double notch, and $K_{\mathrm{t,equi}}$ for approximation by the equivalent notch concept were calculated and compared. In all the cases, $K_{\mathrm{t,exact}}$ can be approximated within ±10% error by $K_{\mathrm{t,double}}$. However, where the ratio of notch root radius was less than 0.1, $K_{\mathrm{t,exact}}$ cannot be approximated by $K_{\mathrm{t,equi}}$ within this error limit.

Keywords: Double notch, stress concentration factor, approximation method

1. Introduction

This study is treated about stress concentration factor, K_{t} of double notch which is cut at both side edge and center of finite plate. In some cases of infinite plate, the stress concentration factor K_{t} of double notch is evaluated by

$$K_{\mathrm{t,\,double}} = K_{\mathrm{t1}}.\,K_{\mathrm{t2}} \quad \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \; (1)$$

where, K_{t1} is the stress concentration factor of the first notch and K_{t2} is the stress concentration factor of second notch. When a small notch is cut at the bottom of first notch of a larger size in an infinite plate as shown in the schematic representation of Fig. 1(a), the stress concentration factor is approximated by double notch concept. On the other hand, when a relatively longer notch is cut at the bottom of another notch, the stress concentration factor K_{t} as shown in Fig. 1(b) is approximated by the equivalent notch concept. In this case, the function of the radius $\rho\,(=R2)$ expresses the stress concentration factor, and equivalent notch length t [1,2]. In this analysis, a several models developed to examine the effect of cutting the second notch at the bottom of the first notch on the variation in the stress concentration factor in a finite plate with both the side edge and center notch. From the computational result, it was found that the accuracy for approximation of base solution by those concepts is dependent on the size and shape of calculation model, for example, the shape of large notch, and the diameter of small notch. Semi-circular and U-shaped notch are used for boundary of model. Three different stress concentration factors $K_{\mathrm{t,exact}}$, $K_{\mathrm{t,double}}$ and $K_{\mathrm{t,equi}}$ were calculated. Then the values of those stress concentration factors were compared. Where, $K_{\mathrm{t,exact}}$ is the stress concentration factor for the original model, $K_{\mathrm{t,double}}$ is for the approximation by the double notch concept, and $K_{\mathrm{t,equi}}$ is for the approximation by equivalent notch concept.

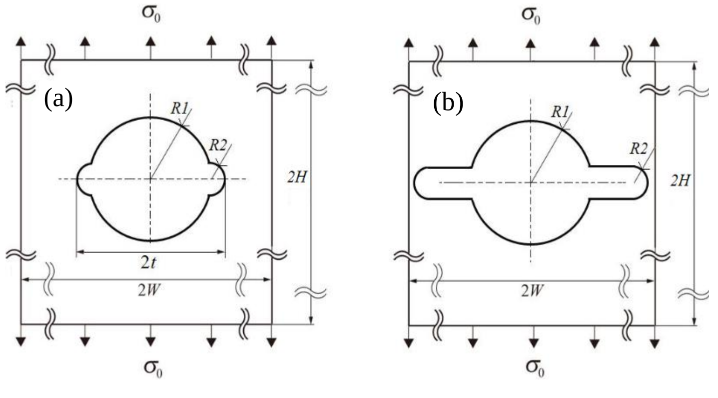

Fig. 1. The shape applied for the concepts of double notch and equivalent notch: (a) approximation in the double notch concept, (b) approximation in the equivalent notch concept.

Also, the variations of stress distribution in front of the notches were investigated by the non-dimensional stress gradient $\sigma/\sigma_{\max}$. Then the stress distribution in front of the notch is calculated to understand the crack initiation and growth behavior.

2. Calculation models and analytical methods

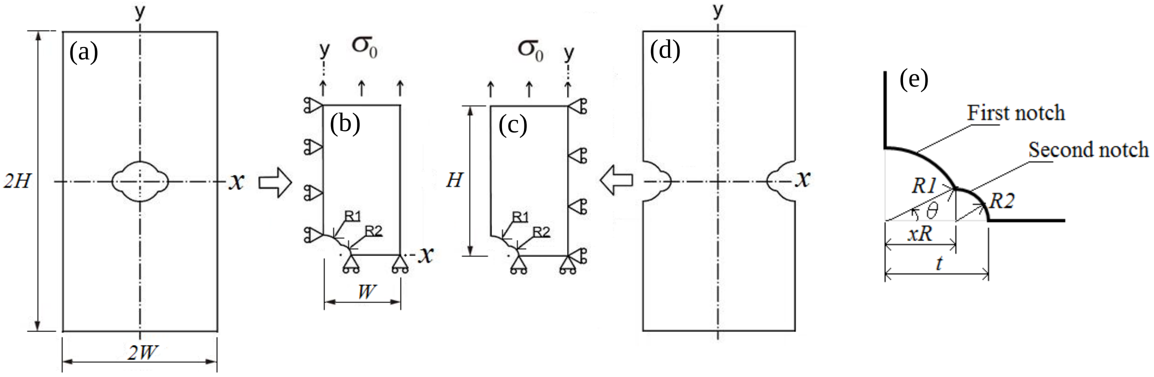

Fig. 2. Geometry of models: (a) Center notch, (b) Calculated model of center notch, (c) Calculated model of side edge notches, (d) Side edge notches, (e) Connecting point of first and second notch ($xR^2 = R_1^2 - R_2^2$)

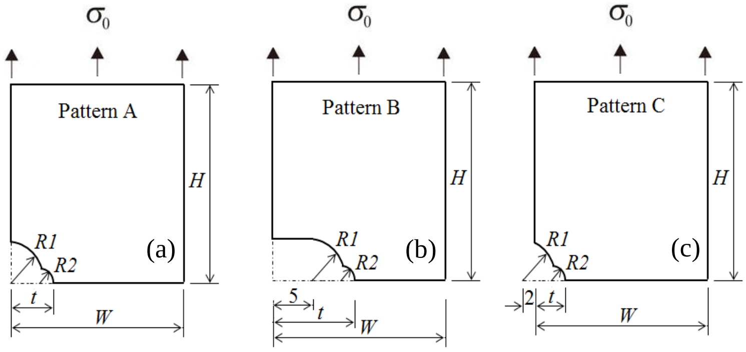

Fig. 3. Analytical model (a quarter of the total shape) used: (a) *W*=10, 20, 80mm, (b) *W*=20mm, (c) *W*=20mm

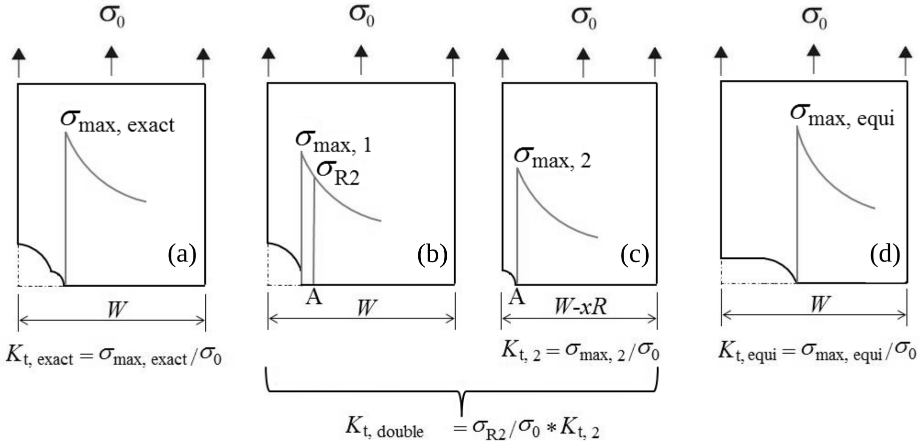

Fig. 4. Evaluation of stress concentration for each case ($K_{t,\,\mathrm{exact}}$; $K_{t,\,\mathrm{double}}$; $K_{t,\,\mathrm{equi}}$): (a) Double notch, (b) First notch, (c) Second notch, (d) U-shaped notch

The geometry of the basic model is shown in Fig. 2. It was a rectangular plate with three symmetrical notches at the center of both *x*-axis and *y*-axis for center notched model, and two symmetrical notches at opposite ends of the *x*-axis and centered along both *x*-axis and *y*-axis for side edge notched model respectively. The calculation was performed by using the body force method [3]. The two-dimensional program was developed by Nisitani and Saimoto [3]. This program was employed in the present calculation, and the calculation was performed with a quarter part of the original model by considering the symmetrical shape. Figure 3 shows schematic representation of distances t and the angle θ between the center of the first notch and intersecting point of the first and second notches. The axial load was applied in this calculation.

The developed models for the computation are shown in Fig. 3. The radius of first notch was 5mm in all cases. On the other hand, the radii of second notch were 0.1mm, 0.3mm, 0.4mm, 0.5mm, 0.7mm, 0.8mm, 1.0mm and 1.4mm respectively for center notched model. For side edge notched model, it was 0.1mm, 0.3mm, 0.5mm, 0.8mm, 1.0mm and 1.4mm, respectively. The basic size of half width W of calculation models was 20mm, and height H was 120mm. The sizes of half width W of pattern A were 10mm, 20mm and 80mm, respectively in three different cases both for the center and the side edge models. The connecting point of first notch and second notch was determined by the distance '*xR'* according to the Fig. 2, and the distance '*xR'* defined by the following equation:

$$xR^2=R1^2-R2^2 \quad \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (2)$$

Two other different patterns B and C were also introduced according to the position of the center of large notch. In case of pattern B for double edge notched models, the center of the first notch is just after 5mm from the corner edge of the original rectangular plate. So, the first notch looks like a U-shape notch for pattern B and for pattern C, the center is 2mm far towards outside from the corner edge of the original plate.

Figure 4 describes how three different kinds of stress concentration factors such as $K_{\mathrm{t,exact}}$; $K_{\mathrm{t,double}}$; and $K_{\mathrm{t,equi}}$ computed by body force method. The value of stress concentration factor that calculated from the applied model is defined as base value, $K_{\mathrm{t,exact}}$. $K_{\mathrm{t,double}}$ is the approximate value with the double notch concept [4]. It is well known that the approximation value by equation (1) gives an over estimation rather than the base value. K_{t1} is the stress concentration factor of the model having only a large size notch ($R1$=5mm), and K_{t2} that for a small size notch but the width in this case is W-xR as shown in Fig. 4(b) and 4(c) respectively. Instead of K_{t1}, we used stress value σ_{R2} at the point A, whose distance from the edge of the original plate is t (=xR+$R2$). Then the $K_{\mathrm{t,double}}$ was approximated as follows:

$$K_{\mathrm{t,\,double}}=(\sigma_{\mathrm{R2}}/\sigma_{\mathrm{o}})*K_{\mathrm{t2}} \quad \dots \dots \dots \dots \dots \dots \dots (3)$$

On the other hand, $K_{\mathrm{t,\,equi}}$ is the approximate stress concentration factor with the equivalent notch concept [1,2]. As shown in Fig. 4(d), the shape of the first notch is ignored and the second notch root radius $R2$ and total half-length t of the notches were used for the calculation of $K_{\mathrm{t,\,equi}}$. The distribution of stress σ in the loading direction in front of the notch was also calculated, and the relationship between the non-dimensional stress $\sigma/\sigma_{\mathrm{max}}$ according to the notch root radius ρ (=$R2$) was examined.

3. Results and discussions

3.1. Comparison of stress concentration for each pattern

The calculations of stress concentration factor in the present models were performed by the equation:

$$K_{\mathrm{t}}=\sigma_{\mathrm{max}}/\sigma_{\mathrm{o}} \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (4)$$

where, σ_{max} is the maximum value of stress at notch bottom and σ_{o} is the stress at upper and lower edges of calculation model. Figures 5-9 show the variation among the various stress concentration factors $K_{\mathrm{t,exact}}$; $K_{\mathrm{t,double}}$; and $K_{\mathrm{t,equi}}$ of both the center notched and side notched models, and the result was compared.

Figures 5-7 show the variation in stress concentration factors for the pattern A with W=10mm, 20mm and 80mm respectively. So, the double notch concept is applied where the size of $R2$ is much smaller than that of $R1$; the calculations were performed where the ratio $R2/R1$ is smaller than 0.3. Figure 5(b) shows the ratio of stress concentration factors. Where, the value of $K_{\mathrm{t,\,double}}/K_{\mathrm{t,\,exact}}$ and $K_{\mathrm{t,\,equi}}/K_{\mathrm{t,\,exact}}$ are 1, the base stress concentration factor can be evaluated by approximation with the double notch or the equivalent notch concept. In the case of pattern A (W=10mm), where the ratio $R2/R1$ is smaller than 0.2 except the value when $R2$=0.1mm for center notched model ($R1$=5mm, $R2$=0.3mm, 0.4mm, 0.5mm, 0.7mm, 0.8mm and 1.0mm respectively), the ratio $K_{\mathrm{t,double}}/K_{\mathrm{t\,,exact}}$ shows a value closer to 1. On the other hand, where $R2/R1$ is larger than 0.05 and less than 0.14, the $K_{\mathrm{t,\,equi}}/K_{\mathrm{t,\,exact}}$ is given close to 1. Also, it is found that $K_{\mathrm{t,\,double}}/K_{\mathrm{t,\,exact}}$ is smaller than 1.1; where $R2/R1$ is within the range of 0 to 0.3 for both center, and double edge notched models, and that $K_{\mathrm{t,\,equi}}/K_{\mathrm{t,\,exact}}$ is also smaller than 1.1 within this range except one value, when $R2$=0.1mm for center notched model and $R2$=0.1mm, 0.3mm and 0.5mm for double edge notched model. Therefore, the double notch and equivalent notch concept can be applied within an error of 10%; where $R2/R1$ is within the range of 0.05 to 0.28 for both the cases, and 0.125 to 0.28 for double edge notched model for approximation by equivalent notch concept. From Fig. 6(b) and 7(b) it was clear that double notch concept within an error of 10% can be applied within the range of 0.02 to 0.28 for both the center and double edge notched models. Also, equivalent notch concept with that error level can be applied when $R2/R1$ is 0.1 to 0.28 for center notched model. But equivalent notch concept cannot be used in the case of Fig. 6(b) for double edge notched model except when $R2/R1$ is greater than 0.25. Also, it was found that the value of $K_{\mathrm{t,\,exact}}$ for pattern A (both for W=20mm and W=80mm) is overestimated when $K_{\mathrm{t,\,equi}}$ are used instead of the $K_{\mathrm{t,\,exact}}$. This kind of approximation should not be applied by $K_{t,\, equi}$ when *R2/R1* is lower than 0.08 for center edge notched model. In this range, the value of $K_{t,\, equi}$ is clearly higher than that of $K_{t,exact}$.

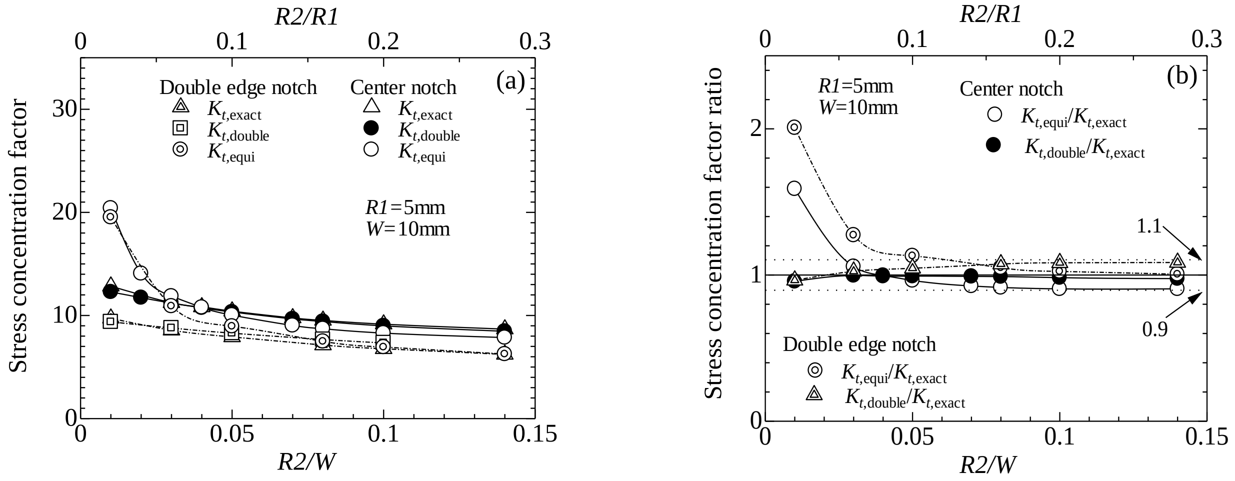

Fig. 5. Variation of stress concentration factors for pattern A with W=10mm:
(a) $K_{t,\, exact}$; $K_{t,\, double}$; $K_{t,\, equi}$ vs. $R2/W$ (b) $K_{t\, equi}/K_{t,\, exact}$ and $K_{t,\, double}/K_{t,\, exact}$ vs. $R2/W$

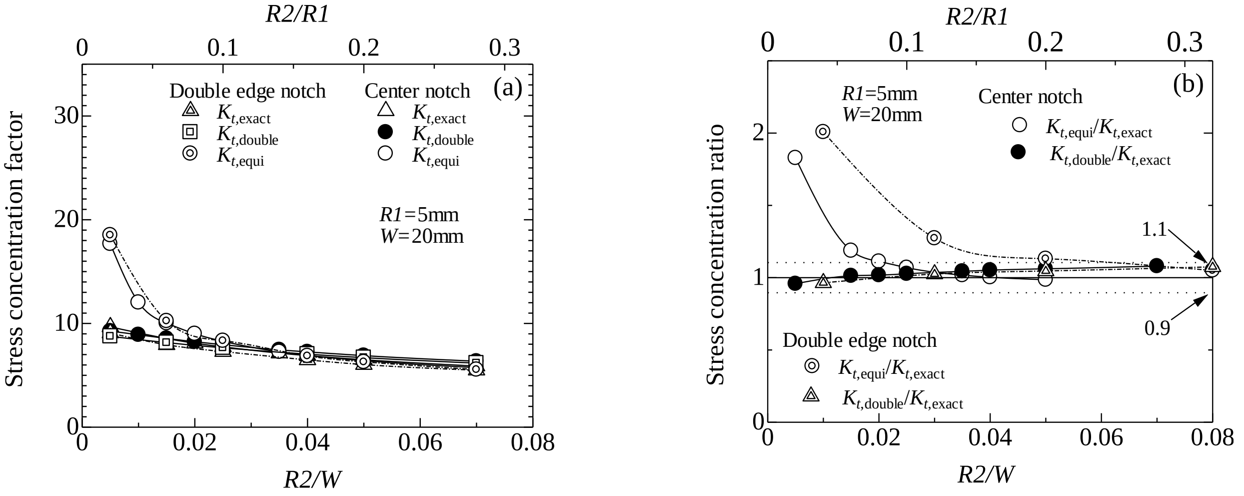

Fig. 6. Variation of stress concentration factors for pattern A with W=20mm:
(a) $K_{t,\, exact}$; $K_{t,\, double}$; $K_{t,\, equi}$ vs. $R2/W$ (b) $K_{t,\, equi}/K_{t,\, exact}$ and $K_{t,\, double}/K_{t,\, exact}$ vs. $R2/W$

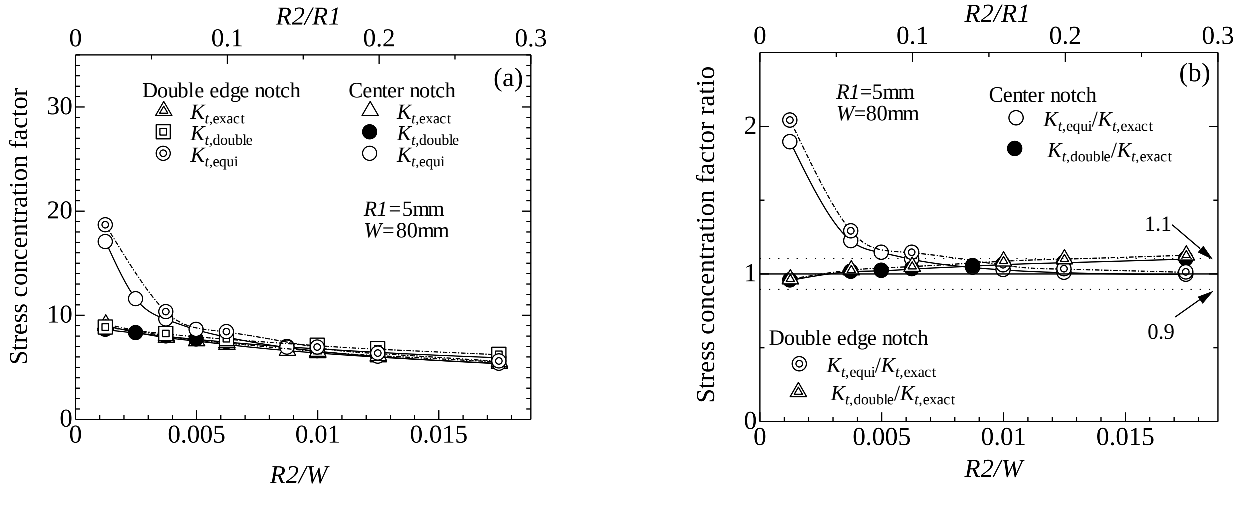

Fig. 7. Variation of stress concentration factors for pattern A with W=80mm:
(a) $K_{\mathrm{t,\,exact}}$; $K_{\mathrm{t,\,double}}$; $K_{\mathrm{t,\,equi}}$ vs. $R2/W$ (b) $K_{\mathrm{t,\,equi}}/K_{\mathrm{t,\,exact}}$ and $K_{\mathrm{t,\,double}}/K_{\mathrm{t,\,exact}}$ vs. $R2/W$

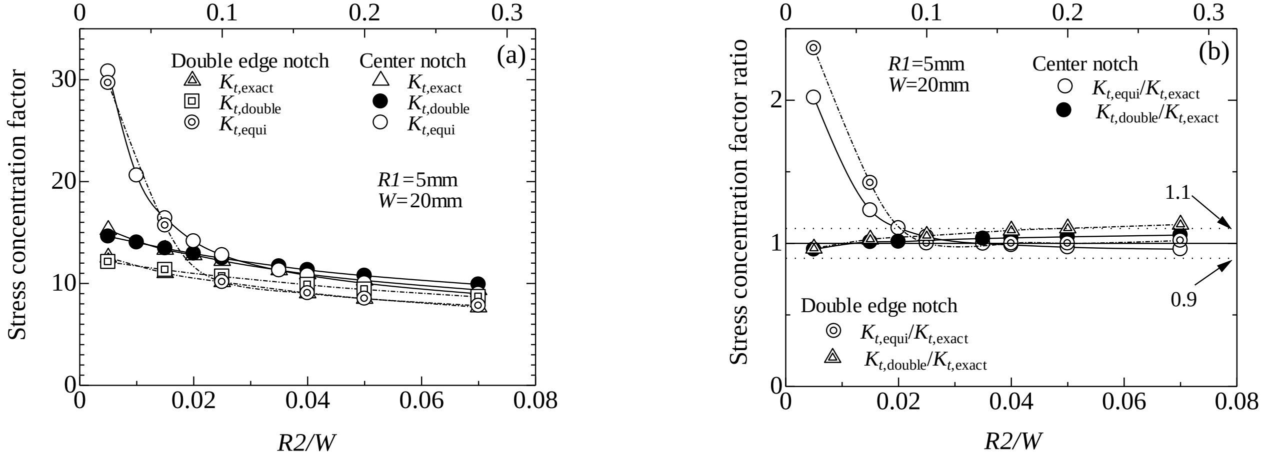

Fig. 8. Variation of stress concentration factors for pattern B with W=20mm:
(a) $K_{\mathrm{t,\,exact}}$; $K_{\mathrm{t,\,double}}$; $K_{\mathrm{t,\,equi}}$ vs. $R2/W$ (b) $K_{\mathrm{t,\,equi}}/K_{\mathrm{t,\,exact}}$ and $K_{\mathrm{t,\,double}}/K_{\mathrm{t,\,exact}}$ vs. $R2/W$

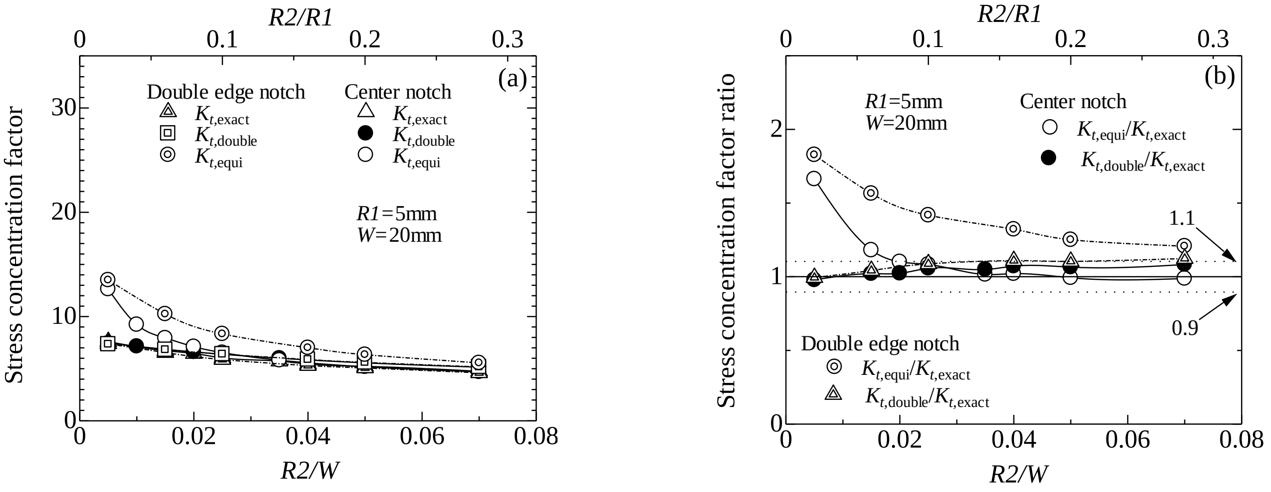

Fig. 9. Variation of stress concentration factors for pattern C with W=20mm:
(a) $K_{\mathrm{t,\,exact}}$; $K_{\mathrm{t,\,double}}$; $K_{\mathrm{t,\,equi}}$ vs. $R2/W$ (b) $K_{\mathrm{t,\,equi}}/K_{\mathrm{t,\,exact}}$ and $K_{\mathrm{t\,,double}}/K_{\mathrm{t\,,exact}}$ vs. $R2/W$

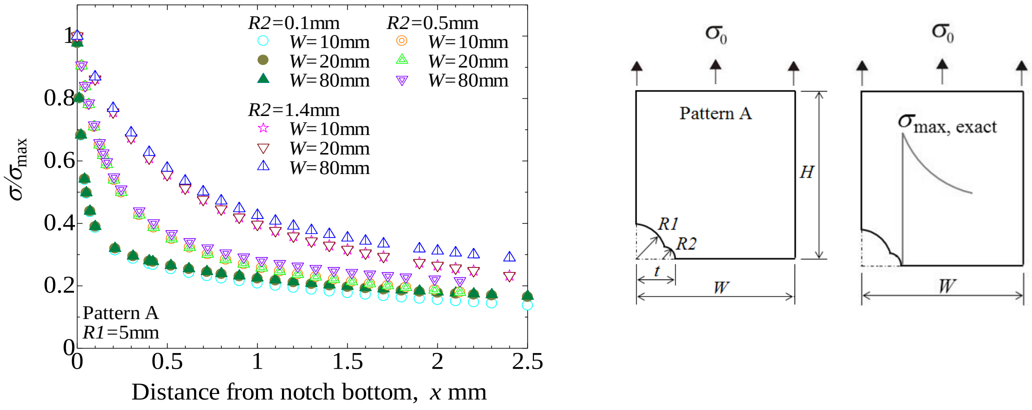

Fig. 10. Stress distributions {Fig. 4(a)} of Pattern A (center notch)

The gradient of stress in the loading direction is an important factor for evaluating the existence of the non-propagating crack at notch root under cyclic loading conditions [1, 2]. Therefore, the gradient of stress in the loading direction was examined. The crack initiation at notch root is related to the maximum value of the stress in the loading direction when an axial cyclic load is applied. The conditions, where the fatigue crack will grow or not, is related to the stress distributions. Figures 10-11 show the non-dimensional stress distribution in front of the notch root in the loading direction. The stress was non-dimensional by the maximum stress at the notch root. Non-propagating crack was expected to be observed in the case of stress distribution of *R2*=0.1mm, not to be of *R2*=1.4mm with center notch pattern.

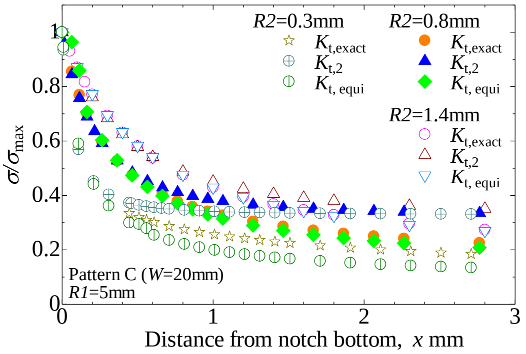

Fig. 11. Stress distributions for patterns with double edge notch which geometries are shown in Fig. 4(a)-4(d)

The stress distribution of σ/σ_{max} is related to the models of Fig. 3 for first notch and purposed notch. Figure 10 shows the comparison of the stress distributions of pattern A in the case of Fig. 3(a) with *W*=10mm, *W*=20mm and *W*=80mm. The distribution of stress σ/σ_{max} is related to the notch root radius *R2* in the present case as well as in the case of the notch in infinite body [4]. Also, it is interesting that the stress distribution for each *R2* value is almost the same among the all patterns of A. From the calculation results of stress distribution, it is found that the gradient of stress σ/σ_{max} near notch root is related to the notch root radius in the present study cases.

Figure 11 shows the variations of σ/σ_{max} where the patterns are different. Where the distance from the notch root is within 0.4mm in all calculated cases, the gradient of stress σ/σ_{max} is almost the same depending on the notch root radius *R2*. Thus, the crack initiation, and growth behavior are expected to be the same within the distance of 0.4 mm from the notch root where the notch root radius *R2* is the same. Therefore, the existence of the non-propagating crack, whose length measured from the notch root is smaller than 0.4mm, can be evaluated by the notch root radius and the maximum value of the stress at notch root. The stress decreased rapidly near the notch root where *R2*=0.3mm. Such kind of stress distribution is related to non-propagating crack behavior [3-5]. For example, the non-propagating cracks in notched specimens of low carbon steel were observed where the notch root radii were lower than 0.5mm [5]. The shape of stress distribution of σ/σ_{max} where *R2*=1.4mm is hardly affected by the shapes of the models of Fig. 4 within the distance of 3mm from notch root. In this case, the gentle decreasing of the stress value from the notch bottom is observed, and the probability of the existence of a non-propagating crack is low. Relationship between the stress distribution near notch root and the crack propagation behavior was discussed by Nisitani [3]. It was shown that the non-propagating crack was not observed at the blunt notch when the stress amplitude of fatigue limit was applied.

4. Conclusions

In this study, approximation technique of stress concentration factor over double notches at both the center, and side edges were examined. The approximation accuracy of stress concentration factor with both the double notch, and equivalent notch concept were computed for both the cases. Within the present study calculation conditions, the value of stress concentration factor of a notched plate can be approximated with an error of 10% by either the double notch or by the equivalent notch concept where radius ratio of *R2*/*R1*=0.1~0.28 [6,7] except the approximation of equivalent notch concept of double edge notch in the case of pattern C and pattern A (*W*=20mm). Also, it is confirmed that the stress distribution near the notch bottom is related to the notch root radius in the present study case.

5. References

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