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Effect of Aspect Ratio on the Micro-scale Responses of Granular Materials

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Abstract

This study presents the effect of aspect ratio on the micro-scale responses of granular materials using the discrete element method (DEM). The ratio of width to height ratio of a particle is referred to as aspect ratio and the variation of aspect ratio changes the shape of particles. Hence, it plays a significant role in the shear characteristics of granular materials. Samples consisting of oval shaped particles with varying aspect ratios were numerically generated using the periodic boundaries. The generated samples were subjected to isotropic compression. The samples were then subjected to biaxial shear under strain controlled conditions. The stress-strain-dilative behavior is significantly influenced by the variations in the aspect ratio of ovals comprising the samples. The evolution pattern of the average and effective coordination number is not so different. Higher the aspect ratio of ovals higher the average and effective coordination number become. The evolution of the slip coordination number depicts a different behavior compared to average and effective coordination number. The evolution pattern of slip coordination number has a bit similarity with the evolution of deviatoric stress. The deviatoric fabric is also influenced by the aspect ratio of particles. The evolution of the deviatoric fabric for strong contacts has strong similarity with that of the stress-strain curve.

Keywords: Aspect ratio, Micro-scale response, Granular materials, Discrete element method.

1. Introduction

Aspect ratio of granular materials plays a significant role in macro- and micro-mechanical characteristics evolve during shear. Aspect ratio (AR) is usually defined as the ratio of width to height of a particle. The higher the aspect ratio the flat the granular material will be. Such characteristics of granular materials have significant impact on the stress-strain-dilative behavior of granular materials, particularly on the position of the peak stress and the value of internal friction angle. Since granular materials are discrete in nature, their mechanical behavior is intricate. There remains many experimentally observed phenomena that are not well understood still today. The internal processes that take place during the laboratory based experiments are not well known even of the use of advanced instrumental facilities such as the photo imaging analysis [1], X-ray tomography [2], wave velocity measurement [3], magnetic resonance imaging [4], etc. These advanced experimental instruments are sophisticated, expensive and time consuming. Besides, it is not possible to extract all the internal data at micro-scale level using these advanced experimental devices or methods. However, the understanding of these micro-processes is essential to develop physically sound constitutive models and to explain the physically observed phenomena from the micro-mechanical point of view. Consequently, a comprehensive study is necessary to understand the micro-mechanical characteristics of a granular system. Since it is difficult to conduct experimental study using the conventional experimental facilities, a good alternative is to adopt any numerical approach that can model the discrete behavior of granular materials. Discrete element method (DEM), pioneered by Cundall and Strack [5], is a numerical method that enables one to model the discrete nature of granular media with varying aspect ratio. Very few studies were reported in the literature that considered the effect of aspect ratio on the micro-scale response of granular materials using the DEM. For example, Nouguier-Lehon et al. [6] considered different aspect ratios (1 to 3) of granular materials and granular material behavior is studied from global and local point of view. In their study, it is noted that the initial anisotropy increases with the aspect ratio of particles and this anisotropy hardly evolves as the aspect ratio of particles increases. Xie et al. [7] studied the influence of aspect ratio on the shear behavior of granular materials and indicated that the particle aspect ratio has a significant influence on the stress-strain curve, peak strength, dilatancy characteristics and critical state behavior. Nevertheless, the samples of these studies yield different initial void ratio prior to shear and consequently, it is difficult to distinguish the effect of aspect ratio from the differences of initial void ratio prior to shear on the stress-strain-dilative behavior. In the present study, samples of same initial void ratio were prepared with different aspect ratios of ovals and the numerical simulation was carried out using DEM to examine the effect of aspect ratio on the micro-scale behavior using the oval shaped particles. An oval is comprised of four pieces of circular arcs joined together. The numerical treatment for ovals was comprehensively described in Kuhn [8]. The schematic diagram of an oval is depicted in Fig.1, where α denotes a splice angle. It should be greater than 0º and no greater than 90º [8]. In Fig. 1, points A1, A2, B1 and B2 indicate the center of the top, bottom, right and left arc, respectively and angle β denotes the orientation angle of oval with the horizontal axis. The height to width ratio (r) of oval must lie within the limits given below:

$$\frac{1-\cos\alpha}{\sin\alpha} \langle r \langle \frac{\cos\alpha}{1-\sin\alpha} \tag{1}$$

To produce the numerical sample, ovals were generated randomly in a rectangular frame with height to width ratio of sample being two. The width to height ratio of all ovals in each sample is 1.25, 1.43 and 1.67, respectively. The dry granular sample was compressed isotropically to 100 kPa and subjected to biaxial shear under the strain control condition. The macroscopic simulated results was compared with the experimental behavior qualitatively. The grain-scale responses evolve during the biaxial shear were precisely monitored and extracted using DEM.

2. Discrete element method

Discrete element method (DEM) is a numerical method proposed by Cundall and Strack [5] to model the discrete nature of granular materials. Each particle in DEM is treated individually. The kinematics of each particle are monitored. Accordingly, each particle can move and rotate through the interactions of the interparticle contacts. The translational and rotational accelerations of a 2D particle in DEM are computed using the Newton's second law of motion as follows:

$$m\ddot{x}_i = \sum F_i \qquad i = 1-2 \tag{2}$$

$$I\ddot{\theta} = \sum M \tag{3}$$

where F_i are the force components, M is the moment, m is the mass, I is the moment of inertia, $\ddot{x}_i$ are the translation acceleration components and $\ddot{\theta}$ is the rotational acceleration of the particle. Velocities and displacements of particles are obtained by integrating the accelerations over time successively. For details of DEM, readers are referred to Cundall and Strack [5]. Computer program OVAL [8], written in FORTRAN language, is used to analyze the particulate assemblies using DEM. It has already been used for many DEM studies so far and its usefulness has been recognized [9-13]. Simple linear contact model consisting of two springs and a friction slider is used.

3. Preparation of numerical sample

A numerical sample consisting of 8192 ovals was prepared. Three different samples were prepared. The aspect ratio of all ovals in each sample was 1.25, 1.43 and 1.67, respectively. Each initial sparse sample was prepared by placing the ovals on equally spaced grid points of a rectangular frame in such a way that the particles have no initial contact. The samples, generated in this way, were compressed isotropically to 100 kPa in different stages with zero interparticle friction coefficient using the periodic boundary, a boundary condition in which the periodic cells are surrounded by the identical cells. Later, interparticle friction coefficient of 0.50 is used during the simulation. After the end of isotropic compression, the void ratio of the numerical samples composed of particles having aspect ratio of 1.25, 1.43 and 1.67, respectively, becomes 0.1214. Three samples were prepared in such a way that the void ratio at the end of isotropic compression becomes same (0.1214).

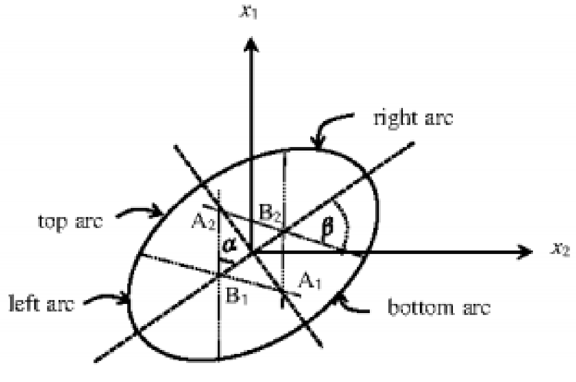

Fig. 1 Composition of an oval shaped particle with reference axes

4. Numerical simulations

Biaxial compression tests were simulated using DEM under the strain control condition. During the shear, the sample height decreased vertically during loading with a very small strain increment of 0.00002% in each step by keeping the stress in lateral direction constant (i.e. 100 kPa). The parameters used in the simulations are presented in Table 1.

Table 1. DEM Parameters used in the simulations

DEM parameters	Value
Normal contact stiffness (N/m)	1×10^{8}
Shear contact stiffness (N/m)	1×10^{8}
Mass density (kg/m^3)	2650
Increment of time step (s)	1×10^{-6}
Interparticle friction coefficient	0.50
Damping coefficients	0.05

5. Stress-strain-dilative response

The relationship between the deviatoric stress, $q = \sigma_1 - \sigma_2$ and the axial strain ε_1 for samples having aspect ratios of 1.25, 1.43 and 1.67, respectively is depicted in Fig. 2. Here, σ_1 is the stress in x_1-direction and σ_2 is the stress in x_2-direction, respectively. The deviatoric stress attains a peak position which is followed by huge strain softening. The position of the strain at which the deviatoric stress peaked varies with the aspect ratio of ovals. Maximum deviatoric stress is observed for sample having aspect ratio of ovals equal to 1.25 and the least deviatoric stress is observed for sample having aspect ratio of ovals equal to 1.67. Sample having aspect ratio of ovals equal to 1.43 depicts the intermediate behavior. Thus, the peak stress decreases with the increases of the aspect ratio of ovals in the samples. This seems opposite to the usual concept that the flat the particles becomes, the higher the interlocking of the particles becomes and consequently, the higher the stress ratio becomes. It should be noted that the initial void ratio of three samples were made equal so that the initial void ratio prior to shear has no effect on the stress-strain behavior, which is different from other studies in the literature. While preparing the isotropic sample, it was noted at the end of isotropic compression that the sample having the aspect ratio of ovals of 1.25 yields the highest void ratio and the sample having the aspect ratio of ovals of 1.67 yields the lowest void ratio. Since the initial void ratio was different, it was difficult to comment that the samples have the same initial densities prior to shear. So, the initial void ratio was made same (i.e. 0.1214) for all the samples and while doing this, the sample having the aspect ratio of ovals of 1.25 may get a bit dense and consequently, the peak stress becomes the maximum for the sample having the aspect ratio of ovals of 1.25. The relationship between the volumetric strain and axial strain is depicted in Fig. 3. The volumetric strain, ε_v, is computed as $\varepsilon_v = \varepsilon_1 + \varepsilon_2$, where ε_1 is the strain in x_1-direction and ε_2 is the strain in x_2-direction, respectively. The positive value of ε_v indicates dilation while the negative value indicates the compression. Sample having aspect ratio of ovals equal to 1.25 depicts huge dilation compared to the sample having aspect ratio of ovals equal to 1.67. The higher deviatoric stress for sample having aspect ratio of ovals equal to 1.25 is related to the higher dilation during the biaxial compression.

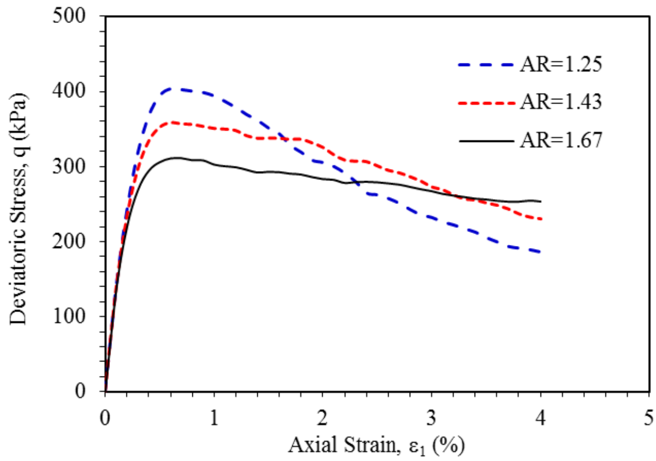

Fig. 2 Stress-strain relationship for the sample having the aspect ratio of ovals of 1.25, 1.43 and 1.67

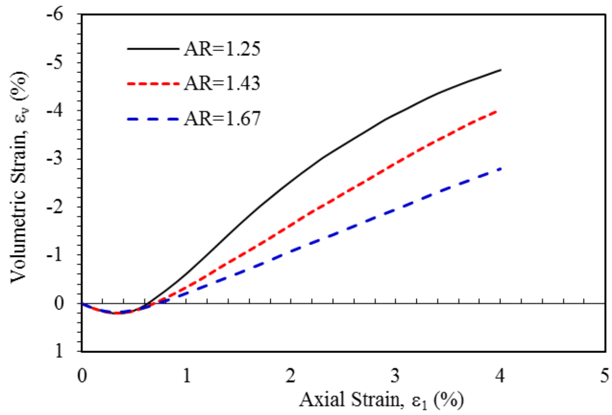

Fig. 3 Dilative response for the sample having the aspect ratio of ovals of 1.25, 1.43 and 1.67

6. Macro-mechanical responses

The evolution of the micro-scale responses such as the average coordination number, effective coordination number, slip coordination number and deviatoric fabric is reported in this section. The average, effective and slip coordination number are defined as $Z = 2N_c / N_p$, $Z_{eff} = 2\overline{N}_c / \overline{N}_p$ and $Z_{sl} = 2N_{sl} / N_p$. Here, N_c , $\overline{N}_c$ and N_{sl} are the total, effective and sliding contacts during the biaxial shear and N_p and $\overline{N}_p$ is the number of total and effective particles. The evolution of the average coordination number, effective coordination number, slip coordination number is depicted in Figs. 4, 5 and 6, respectively. Aspect ratio has significant effect on the evolution of the average coordination number, effective coordination number and slip coordination number. The evolution pattern of the average and effective coordination number is not so different. However, the highest values of average and effective coordination number are noticed for sample having aspect ratio of ovals equal to 1.67 than that of 1.25. This is opposite to what is noticed during the evolution of the stress-strain response. This is because, when the aspect ratio increases, the particles become flat and consequently increases the chance of more contacts with their neighbor particles due to the particle geometry. As a result, the coordination number for sample having aspect ratio of ovals equal to 1.67 is a bit elevated compared to that of 1.25. The evolution of the slip coordination number on the other hand depicts a different behavior compared to average and effective coordination number. The slip coordination number gradually increases with the increase of strain and then it is followed by huge reduction. The evolution pattern has a bit similarity with the evolution of deviatoric stress. It should be noted that the highest values of slip coordination number is noticed for sample having aspect ratio of ovals equal to 1.67 than that of 1.25. The evolution of the deviatoric fabric with strain is depicted in Fig. 7. The deviatoric fabric is defined as follows [12]: $H_d^s = H_{11}^s - H_{22}^s$, where $H_{ij}^s = \dfrac{1}{N_c}\sum_{\alpha=1}^{N_c^s} n_i^\alpha n_j^\alpha$ Here, n_i^α is the unit contact normal vector of the $\alpha-$th strong contact and N_c^s is the number of strong contacts. It is noted that the evolution of the deviatoric fabric for strong contacts has strong similarity with that of the stress-strain curve. The deviatoric fabric is also influenced by the aspect ratio of particles.

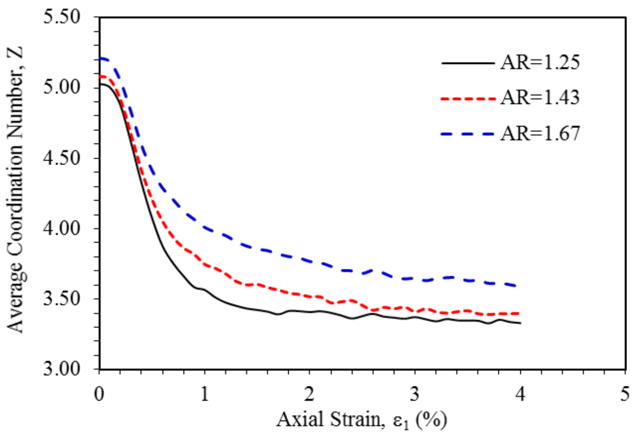

Fig. 4 Evolution of Z for the sample having the aspect ratio of ovals of 1.25, 1.43 and 1.67

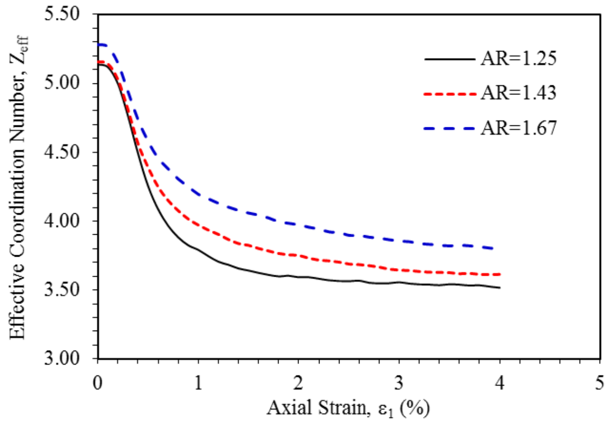

Fig. 5 Evolution of Z_{eff} for the sample having the aspect ratio of ovals of 1.25, 1.43 and 1.67

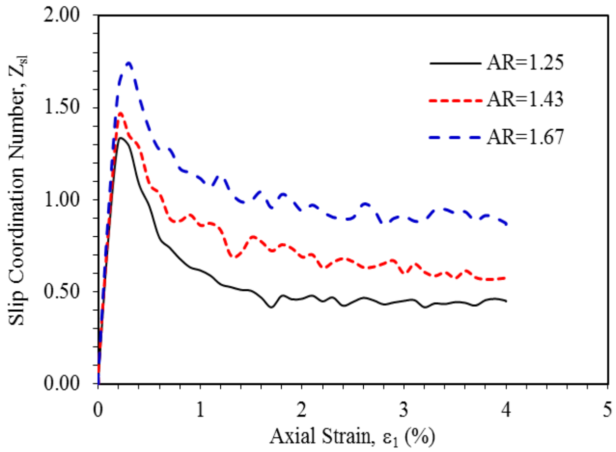

Fig. 6 Evolution of Z_{sl} for the sample having the aspect ratio of ovals of 1.25, 1.43 and 1.67

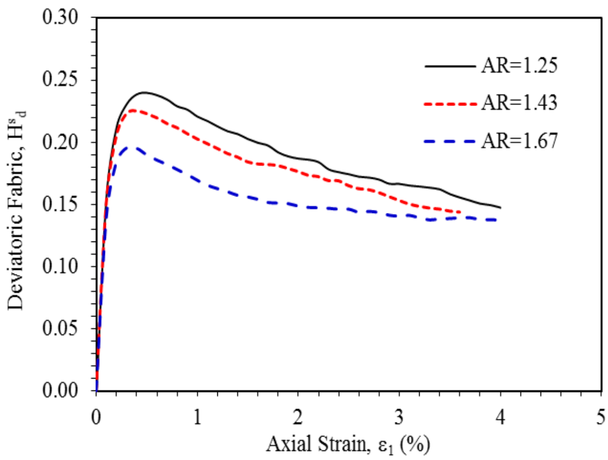

Fig. 7 Evolution of H_d^s for the sample having the aspect ratio of ovals of 1.25, 1.43 and 1.67

7. Conclusions

This study summarizes the effect of aspect ratio of oval shaped particles on the micro-scale behavior during the biaxial shear using the DEM. It is noted that the stress-strain-dilative behavior is significantly influenced by the aspect ratio of oval shaped particles. The findings of the study is presented as follows:

(i) Aspect ratio has significant influence on the evolution of the average coordination number, effective coordination number and slip coordination number.
(ii) The evolution pattern of the average and effective coordination number is not so different. However, the highest values of average and effective coordination number are noticed for sample having aspect ratio of ovals equal to 1.67 than that of 1.25.
(iii) The evolution of the slip coordination number depicts a different behavior compared to average and effective coordination number. The slip coordination number gradually increases with the increase of strain and then it is followed by huge reduction.
(iv) The evolution pattern of slip coordination number has a bit similarity with the evolution of deviatoric stress. The highest values of slip coordination number is noticed for sample having aspect ratio of ovals equal to 1.67 than that of 1.25.
(v) The deviatoric fabric is also influenced by the aspect ratio of particles. The evolution of the deviatoric fabric for strong contacts has strong similarity with that of the stress-strain curve.

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