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Measurement of Small Strain Stiffness Parameters of Granular Materials by DEM

Md. Mahmud Sazzad[1], Md. Shariful Islam[2] and Md. Sumon Hossain[3]

[1]Department of Civil Engineering, RUET, Bangladesh, *E-mail:mmsruet@gmail.com*
[2]Department of Civil Engineering, RUET, Bangladesh, *E-mail: sharif.ruet12@gmail.com*
[3]Department of Civil Engineering, RUET, Bangladesh, *E-mail: sumonhossain1264@gmail.com*

Abstract

Small strain stiffness is very important in the numerical analysis of granular materials. The prediction of the behavior of granular materials depends on the accuracy of the determination of the small strain stiffness. It is not easy to compute the small strain stiffness of granular materials experimentally because the measurement of deviatoric stress at a very small strain is very subtle and sophisticated. In this paper, the stiffness of granular materials at a very small strain level (less than 0.001% strain) is measured using the discrete element method (DEM). A numerical sample consisting of 9826 spheres similar to dry grade chrome steel balls was prepared numerically and subjected to shear. The stress-strain behavior obtained from the numerical simulation was compared with the experimental results and then the small strain stiffness was computed. It is noted that DEM can successfully measure the small strain stiffness of granular materials.

Keywords: Small strain stiffness, granular materials, discrete element method.

1. Introduction

To predict the short-term residual deformation of the ground and structural displacement, the deformation characteristics of granular materials at small strain level play a very significant role. Experimentally, the deformation characteristics of granular materials at small strain level can be evaluated using static and dynamic methods. The bender element test was widely used to measure these characteristics dynamically [1]. Apart from this method, a technique using accelerometers and wave sources was recently introduced for dynamic measurement [2]. Again, small strain stiffness can be estimated by applying many small unload/reload cycles of axial stress statically in the laboratory tests within a very small strain range smaller than 0.001% [3]. It should however be remembered that estimation of small strain stiffness at a very small strain range requires the experimental devices to be very precise. Conventional experimental facilities may not be able to measure the strain at such a very small strain level. Numerical methods such as the discrete element method (DEM) pioneered by Cundall and Strack [4] can be a nice alternative. So far, very few studies were reported in the literature that considered the DEM to estimate the small strain stiffness parameters. The aim of the present study is to simulate the laboratory stress-strain response of conventional triaxial tests carried out using the steel balls (i.e. spheres) and then measure the small strain stiffness parameter such as the initial Young's modulus of elasticity. To do this, the laboratory based conventional triaxial (CTC) test reported in Cui et al. [5] and O'Sullivan et al. [6] on dry grade chrome steel balls under vacuum confinement of 80 kPa was simulated using the discrete element method. A numerical sample consisting of 9826 spheres was randomly generated without any initial interparticle contact. The generated numerical sample was compressed in different stages to attain a confining pressure of 80 kPa. During the compression, the periodic boundary condition was applied. The material properties of the laboratory samples and DEM parameters used in the simulation were kept same. The simulation of the CTC test was carried out under the strained controlled condition using DEM. A very small strain increment was assigned so that the quasi-static condition could be attained and the effect of numerical damping became minimum. The simulated data were recorded at desired intervals for the post analysis. The data at a very small strain range smaller than 0.001% was carefully observed and small strain stiffness parameters were measured.

2. Discrete element method

Discrete element method (DEM) is a numerical method [4] to model the discrete behavior of granular materials. Each particle in discrete element modeling can move and rotate through the interactions of the interparticle contacts. The translational and rotational accelerations of a 3D particle in DEM are computed using the Newton's second law of motion and are expressed as follows:

$$m\ddot{x}_i = \sum F_i \qquad i = 1-3 \tag{1}$$

$$I\ddot{\theta} = \sum M \tag{2}$$

where F_i are the force components, M is the moment, m is the mass, I is the moment of inertia, $\ddot{x}_i$ are the translation acceleration components and $\ddot{\theta}$ is the rotational acceleration of the particle. Velocities and displacements of particles are obtained by integrating the accelerations over time successively. For basics of DEM, readers can refer to Cundall and Strack [4]. Computer program OVAL [7] is used to analyze the particulate assemble using DEM. The effectiveness of OVAL has already been recognized [8-11]. In the present study, Hertz-Mindlin contact model is used. The normal force of a Hertz-type contact is computed as follows [12]:

$$F^n = \frac{\bar{E}a^3}{R} \tag{3}$$

$$\bar{E} = \frac{8G}{3(1-\nu)} \tag{4}$$

$$a = \sqrt{\frac{d \times R}{2}} \tag{5}$$

$$R = \frac{2R_1R_2}{R_1 + R_2} \tag{6}$$

Here, $\bar{E}$ is the elastic constant, a is the contact radius, d is the overlap between the contacting particles, R is the effective radius of curvature, R_1 and R_2 are the radii of curvatures of two particles at contact.

3. Preparation of numerical sample

In the present study, the laboratory based triaxial test (CTC test) reported in Cui et al. [5] and O'Sullivan et al. [6] on dry Grade chrome steel balls under vacuum confinement of 80 kPa was used to compare with that of DEM to depict the authentication of DEM based simulated results. Three types of spheres having the radii of 2 mm, 2.5 mm and 3 mm, respectively having a mixing ratio of 1:1:1 with a sample height to width ratio of two were used in the laboratory test. The void ratio of nonuniform sample became 0.603. The characteristics of the spheres used in the CTC tests as reported in Cui et al. [5] and O'Sullivan et al. [6] are given in Table 1. A numerical sample consisting of 9826 spheres was randomly generated without any initial contacts. The spheres were modeled as the particles. This is because it simplifies the contact detection among spheres and therefore, it reduces the computational cost of the simulation. Moreover, the primary goal of the present study is to compare the simulated results of virtual triaxial test with the similar laboratory based CTC test reported in Cui et al. [5] and O'Sullivan et al. [6]. The radii of spheres used in the numerical samples were 2 mm, 2.5 mm and 3 mm, respectively with a mixing ratio of 1:1:1, as has been reported in Cui et al. [5] and O'Sullivan et al. [6]. The initial sample is surrounded by the periodic boundaries, a boundary condition in which the periodic cells are surrounded by the identical cells. The generated numerical sample was compressed in different stages to attain a confining pressure of 80 kPa. The consolidation of the initially generated sparse sample was carried out using the strain controlled condition. After the end of isotropic compression, the void ratio of the numerical sample became 0.626.

Table 1. Characteristics of dry grade chrome steel balls used in the CTC test (after Cui et al. [5])

Properties	**Values**
Density of spheres (kg/m^3)	7.8×10^3
Shear modulus (Pa)	7.9×10^3
Poisson's ratio	0.28
Interparticle friction coefficient	0.096
Boundary friction coefficient	0.228

4. Numerical simulations

The simulation of the CTC test was carried out under the strained controlled condition using DEM. A very small strain increment was assigned so that the quasi-static condition could be attained and the effect of numerical damping became minimum. To monitor the quasi-static condition, the non-dimensional index is defined as follows [13]:

$$I_{uf} = \sqrt{\frac{\sum_{1}^{N_p} F_{ubf}^2 / N_p}{\sum_{1}^{N_c} F^2 / N_c}} \times 100\,(\%) \,, \tag{7}$$

where F_{ubf}, F, N_p and N_c denote the unbalanced force, contact force, number of particles and number of contacts, respectively. Index I_{uf} is directly related to the accuracy of the simulation. Lower the value of I_{uf}, higher the accuracy of the simulation. The material properties and DEM parameters used in the simulation are shown in Table 2. Note that the material properties of the DEM simulation are same as that of grade chrome steel balls used in the CTC test.

Table 2. Material Properties and DEM parameters used in the present study

Properties	**Values**
Density of spheres (kg/m^3)	7.8×10^3
Shear modulus (Pa)	7.9×10^3
Poisson's ratio	0.28
Interparticle friction coefficient	0.096
Increment of time step (s)	1.0×10^{-6}
Damping coefficients	0.10

5. Stress-strain response

The simulated stress-strain behavior is compared with the laboratory based CTC test as reported in Cui et al. [5] and depicted in Fig 1. The normalized deviatoric stress is defined here as $\sigma_d = (\sigma_1 - \sigma_3)/\sigma_3$, σ_1 and σ_3 are the stresses in vertical and lateral directions, respectively. The stress-strain behavior by DEM has excellent agreement with the laboratory based CTC test reported in Cui et al. [5]. This quantitative validation of the simulated results with that of the experiment depicts the versatile nature of the present study by DEM and proofs that DEM can successfully replicate the behavior of granular materials quantitatively.

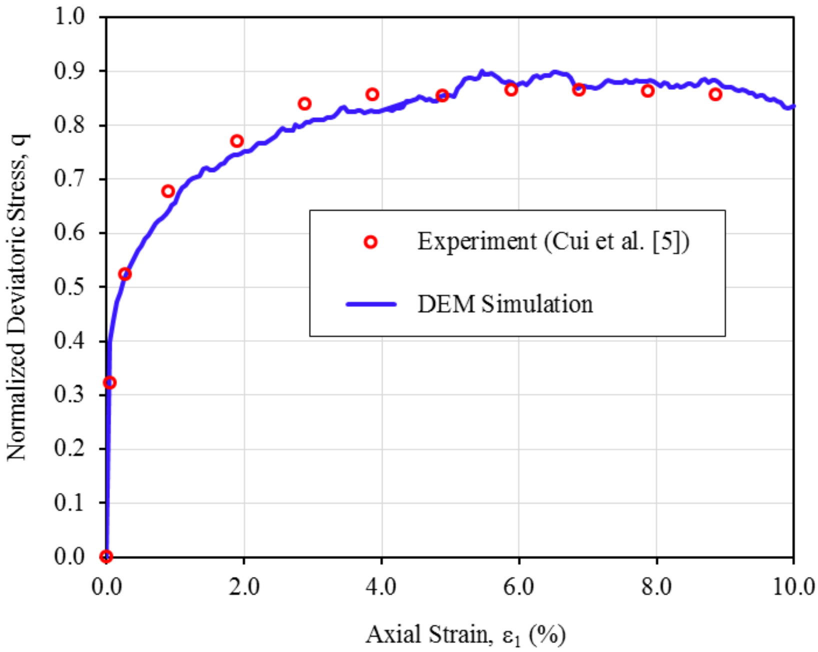

Fig. 1 Comparison of the simulated stress-strain behavior using DEM with the laboratory based CTC test reported in Cui et al. [5]

6. Measurement of small strain stiffness parameter

The small strain stiffness parameter such as the Young's modulus was measured at a very strain range (smaller than 0.001%). To do so, the simulated data were recorded at each time step. The relationship between the deviatoric stress and strain at a very small range is depicted in Fig. 2. Please note that the relationship between the deviatoric stress ($\sigma_1 - \sigma_3$) and axial strain is linear at a very small strain range during loading. Similar linear relationship was also noticed in the experimental studies in the literature [e.g., 13]. The initial value of Young's modulus, E obtained from Fig. 2 is 423 MPa. The quantitative measure of stiffness parameters such as the Young's modulus by DEM depicts the ability of DEM simulation to capture the small strain stiffness parameters.

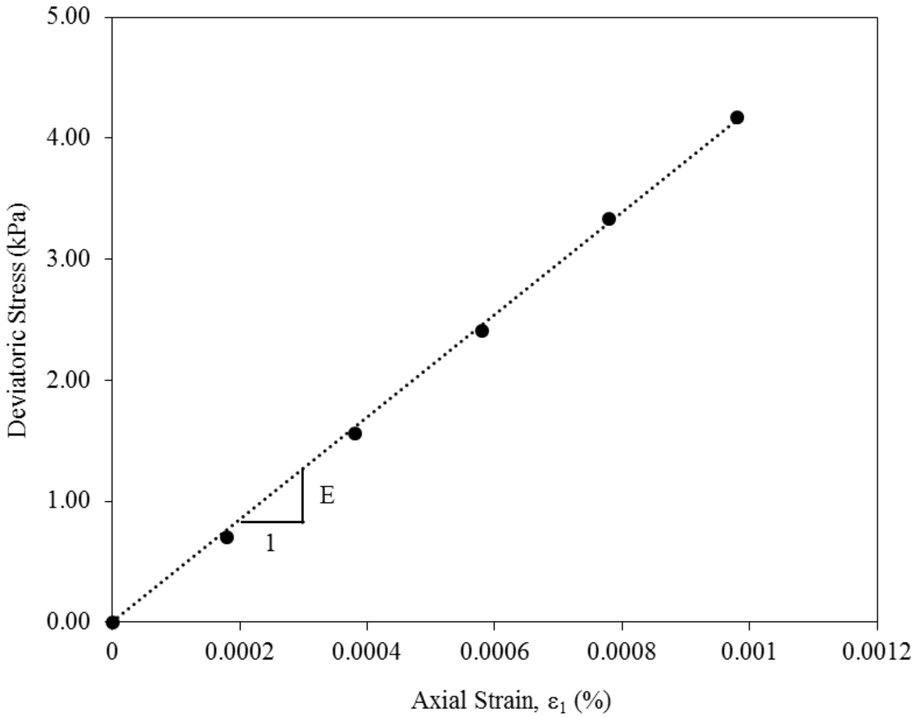

Fig. 2 Stress-strain relationship obtained from DEM simulation at a very small stain range smaller than 0.001%

7. Conclusions

This study presents the simulation of CTC test by DEM and compares the simulated results with the laboratory based experimental study of CTC test on dry grade chrome steel balls under vacuum confinement of 80 kPa. The simulated stress-strain behavior depicts an excellent agreement with the experimental stress-strain behavior. The small strain stiffness parameter such as the Young's modulus is measured. It is noted that the stress-strain behavior at small strange smaller than 0.001% is linear. It illustrates the capability of DEM to measure the small strain stiffness parameter of an assembly of spheres.

8. References

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