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Minimizing Make span for Flow Shop Scheduling using Matrix Manipulation and Heuristic methods under processing uncertainty

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Abstract

Scheduling has to consider operations sequences, machine load and availability of machines. Its aim is to optimize the objectives with the allocation of resources to tasks within the given time periods. A typical flow shop scheduling problem involves the determination of the order of processing of jobs with different processing times over different machines. In reality, any inaccuracy in scheduling can cause long lead-time, production cost increase, and lateness. This inaccuracy may occur due the inaccurate information, uncertainties in demand and uncertainties production facilities. In this paper, matrix manipulation method with MATLAB is proposed to solve flow shop scheduling problem of n jobs on m machines under uncertain processing time. The problems have been considered for comparative analysis with Palmer's heuristic, CDS heuristic & NEH heuristic. The preliminary result indicates that the proposed code is very efficient and time saving in comparison with other methods to find out the minimum makes span through an optimal sequence for flow shop scheduling problem of n jobs on m machines.

Keywords: Scheduling, Flow shop, Uncertainty, Make span, MATLAB, Heuristic.

1. Introduction

Scheduling plays an important role in most manufacturing and service systems. Scheduling is a decision making practice that is used on a regular basis in many manufacturing and services industries. Its aim is to optimize one or more objectives with the allocation of machines to tasks over given time periods. Flow shop Scheduling is used to determine the optimal sequence of n jobs to be processed on m machines in the same order. In this paper, the problem considered in which processing times of jobs are uncertain. The uncertain parameters are represented by triangular fuzzy number. We have used Liou and Wang [1] approach for ranking fuzzy numbers to precisely determine the total integral value of triangular fuzzy number with respect to the degree of achievement. In this paper an attempt was made to apply matrix manipulation method in MATLAB for finding out the best optimal result for the flow shop scheduling problem without performing any manual calculation at very minimum effort compared to other heuristic methods.

2. Literature Review

Most research works for minimizing make span in flow shop scheduling have been being done over the last half centuries. From then many methods have developed. One of them was Johnson [2] who proposed an algorithm to solve general flow shop problem for two machines and n jobs to find out the optimum sequence. After that Johnson's algorithm was extended for three machines and n jobs. Palmer [3] proposed an algorithm which is a slope indexing method to solve n jobs and m machines for finding optimal sequence. Campbell et al. [4] proposed an algorithm which was an extension of Johnson's algorithm for solving flow shop scheduling problems to find optimal sequence with minimum make span. Gupta [5] developed an algorithm which was similar to Palmer's algorithm in which the slope index was defined in a different way by considering some interesting facts of Johnson's algorithm for three machines. Dannenbring [6] developed a 'Rapid Access' procedure which combined the advantages of Palmer's and CDS algorithm to provide a better solution in which it solves only one artificial problem using Johnson's algorithm instead of solving m-1 artificial two machine problems. Nawaz et al. (NEH) [7] proposed an algorithm which is based on the assumption of assigning the highest priority of job with higher total processing time on all machines. It is a constructive algorithm in which sequences are built by adding a new job at each step and finding the optimal solution. Rajendran [8] proposed a method for solving flow shop scheduling problem with multiple objectives of optimizing make span, total flow time and idle time for machines in which the first sequence was taken from CDS algorithm. Tejpal & Jayant

[9] proposed a method which is an extension of Palmer's heuristic for the flow shop scheduling problem. Tang and Zhao [10] developed a model for scheduling a single semi continuous batching machine for scheduling jobs on the machine to minimize make span. Eren and Guner [11] developed a bi-criteria flow shop scheduling problem with a learning effect in a two-machine flow shop for finding a sequence that minimizes a weighted sum of total completion time and make span. Fuzzy sets are often used to describe the processing time when the processing time is uncertain. A number of fuzzy approaches to flow shop scheduling problems have been established. The job sequencing with fuzzy processing time was addressed by MacCahon and Lee [12]. Hong and Chuang [13] proposed a new triangular fuzzy Johnson algorithm. Seyed Reza Hejari [14] etc introduced an improved version of Mc Cahon and Lee algorithm. Yager [15], Shukla and Chen [16], Marin and Roberto [17] have contributed remarkably in the field of flow shop scheduling problems. Ishibuchi and Lee [18] formulated the fuzzy flow shop scheduling problem with fuzzy processing time.

3. Treatment of the fuzzy variables

This work assumes that the decision maker (DM) has already adopted the pattern of triangular possibility distribution to represent the crashing cost, variable indirect costs per unit time and Available total budget in the original fuzzy linear programming problem. In the process of defuzzification, this work applies Liou and Wang (1992) approach for ranking fuzzy method to convert fuzzy number into a crisp number.

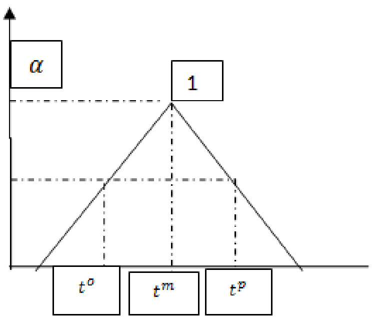

Figure: Membership function of t^α

If the minimum acceptable membership level α, then corresponding auxiliary crisp of triangular fuzzy number $t = [t^o , t^m , t^p]$ is:

$t^\alpha = ½\{\alpha t^o + t^m + (1-\alpha)t^p\}$…………(1)

The primary advantages of the triangular fuzzy number are the simplicity and flexibility of the fuzzy arithmetic operations. For instance, Figure 1 shows the distribution of the triangular fuzzy number of t. In practical situations, the triangular distribution of t may: (1) the most pessimistic value (t^p) that has a very low likelihood of belonging to the set of available values (possibility degree 0 if normalized); (2) the most likely value (t^m) that definitely belongs to the set of available values (membership degree = 1 if normalized); and (3) the most optimistic value (t^o) that has a very low likelihood of belonging to the set of available values (membership degree = 0 if normalized).

4. Data collection & case description

We have collected data of 4 Jobs, 10 Machines flow shop problem with processing times in second described by triangular fuzzy numbers as given in table 1. The given data is collected from RFL Plastics Ltd, 105 Pragati Sarani, Middle Badda, Dhaka 1212, which is a leading manufacturing company in Bangladesh. They produce Premium/Elegant/Fancy Towel Hanger (J1), Premium Tissue Holder (J2), Premium/Fancy Soap case(J3), Elegant Corner Rack (J4) and for making these they need six machines namely Sealing (M2), lock setting (M3), Body setting (M4), Royal Plug packing (M5) and Finished Good Packing machines (M6).They produce these products as J1-J2-J3-J4 sequence to M1-M2-M3-M4-M5-M6 order machine sequence.

Table 1: Fuzzy processing times

JOB	M1	M2	M3	M4	M5	M6
J1	(21.5,22,23)	(23,24,25.5)			(15.5,16,17.5)	(15,16,16.5)
J2		(23.5,24,25)	(24,25,26.5)	(27,28,29.25)	(34.20,35,36.75)	
J3	(9.6,10,11.1)		(11.2,12,12.9)		(14.3,15,16.2)	(11.35,12,12.9)
J4		(20.95,22,23.5)		(31.2,32,33.4)	(20.95,22,23.5)	(20.4,21,22.10)

In that company all the jobs need to have same operation for meeting customer demand as product. So as the position of machines can always be fixed. And that is why this is flow shop scheduling and will be fruitful to our selected methodologies. By this way, they usually face high time to the get the final products. Here we find the problem and take challenge to reduce the total completion time. In the process of defuzzification, this work applies Liou and Wang (1992) approach for ranking fuzzy method to convert fuzzy number into a crisp number. If the minimum acceptable membership level $\alpha = 0.5$, then corresponding auxiliary crisp of triangular fuzzy number using equation (1) is given in the table 2 and table 3.

Table 2: Crisp value of Fuzzy processing times when $\alpha = 0.5$

JOB	M1	M2	M3	M4	M5	M6
J1	22.125	24.125	0	0	16.25	15.875
J2	0	24.125	25.125	28.0625	35.2375	0
J3	10.175	0	12.025	0	15.125	12.0625
J4	0	22.025	0	32.15	22.1125	21.125

Table 3: Crisp value of Fuzzy processing times when $\alpha = 0.6$

JOB	M1	M2	M3	M4	M5	M6
J1	22.2	24.25	0	0	16.35	15.95
J2	0	24.2	25.25	28.175	35.365	0
J3	10.25	0	12.11	0	15.22	12.14
J4	0	22.16	0	32.26	22.24	21.21

5. The solving approaches and procedures

The matrix manipulation method using MATLAB, Palmer's heuristic, CDS heuristic and NEH algorithm are used and compared for finding out the optimal result for the collected flow shop problem. Palmer's heuristic have the calculation of slop of the completion times of each job respected to the machines and with decreasing order of it makespan is being determined. It has limitation of much number products and machines production scheduling problem. CDS heuristic and NEH algorithm are good compared to Palmer's heuristic because of in there the makespan are to calculate for more than one job sequence. And in MATLAB manipulation, the makespan for the jobs of every possible job sequences. Since the problem is of flow shop production scheduling, the heuristic algorithms and our proposed MATLAB coding are assembled here to find out optimal makespan.

Matrix Manipulation in MATLAB

MATLAB (matrix laboratory) is a multi-paradigm numerical computing environment and fourth-generation programming language. A MATLAB code has been developed using matrix manipulation method to find out the optimal result for the given problem. In our MATLAB code, permutation system has been used where every possible permutation of the jobs is being included as well their makespans and it will also find out what the minimum completion time is and also its job sequence. With increasing but a certain risk factor α, it has been seen that the completion time is also increased. That's why the company gets more chances to cope up with uncertain time and to get their final product within lead time. The outputs derived from the code are given below.

Table 4: Final result for matrix manipulation in MATLAB

Type of Processing Times	OUTPUT
Processing times when $\boldsymbol{\alpha}$=0.5	Optimal Time=158.80s, Optimal Sequence= J4-J1-J3-J2
Processing times when $\boldsymbol{\alpha}$=0.6	Optimal Time=159.44s, Optimal Sequence= J4-J1-J3-J2

Palmer's Heuristic

In flow shop scheduling, Palmer proposed a heuristic to minimize the make-span measure. He mainly proposed a slope index S_j for each job. The formula for the slope index S_j is shown below.

$$S_j = (m-1)\, t_{j.m} + (m-3)\, t_{j.m-1} + (m-5)\, t_{j.m-2} + \ldots - (m-3)\, t_{j.2} - (m-1)\, t_{j.1}$$

Where m is the total number of machines.

Procedure:

Step 1.Compute slope for each job.

Step 2.Arrange the jobs as per the decreasing order of slope.

When **α**=0.5 then the company gets more time than previous one and for considering the value of risk factor a little bit uncertainty of time can be minimized easily. And for 0.5 value of the risk factor Palmer’s heuristic give the J4-J3-J2-J1 job sequence. This job sequence shows total job completion time 166.7 seconds which is less than traditional completion time of the company.

Table 5: Solution of Palmer’s heuristic with J4-J3-J2-J1 when **α**=0.5

Job	M1	M2	M3	M4	M5	M6
J4	0	22.025	22.025	54.175	76.2875	97.4125
J3	10.175	22.025	34.05	54.175	91.4125	109.475
J2	10.175	46.15	71.275	99.375	134.575	134.575
J1	32.3	70.275	71.275	99.375	150.825	166.7

And in second case, for 0.6 value of the risk factor, Palmer’s heuristic gives a higher completion time than **α**=0.5 with job sequence J4-J3-J2-J1 and the total completion time is 167.45 seconds. In this sense, the company gets good opportunity to cope up with uncertain processing times of each job.

Table 6: Solution of Palmer’s heuristic with J4-J3-J2-J1 when **α**=0.6

Job	M1	M2	M3	M4	M5	M6
J4	0	22.16	22.16	54.42	76.66	97.87
J3	10.25	22.16	34.27	54.42	91.88	110.01
J2	10.25	46.36	71.61	99.785	135.15	135.15
J1	32.45	70.61	71.61	99.785	151.5	167.45

CDS Heuristic

CDS heuristics is basically an extension of the Johnson's algorithm. The main objectives of the heuristic are the minimization of make-span for n jobs and m machines in a deterministic flow shop scheduling problem. The CDS heuristic forms in a simple manner a set of an m-1 artificial 2-machine sub problem for the original m machine problem by adding the processing times in such a manner that combines M1, M2,...,Mm-1 to pseudo machine 1 and M2, M3,... M_m to pseudo machine 2. Finally, by using the Johnson's 2-machines algorithm each of the 2-machine sub-problems is then solved. The best of the sequence is selected as the solution to the original m-machine problem. For the given flow shop problem as stated in table 1 of size 6×4 using this heuristic the following sequences and make span has been established.

Table 7: Solution of CDS heuristic with J4-J3-J2-J1 when **α**=0.5

Job	M1	M2	M3	M4	M5	M6
J4	0	22.025	22.025	54.175	76.2875	97.4125
J3	10.175	32.15	44.175	54.175	91.4125	109.475
J1	20.3	56.275	56.275	56.275	107.6625	125.35
J2	20.23	80.4	105.525	133.5875	168.825	168.825

When **α**=0.5 applying CDS heuristic it is observed that the sequence J4-J3-J1-J2 will have minimum make span and the total completion time is 168.825 seconds. The makespan from CDS heuristic is higher than Palmer’s heuristic with same uncertain processing times of the jobs. For this the result given by the CDS heuristic is obsolete by the Palmer’s heuristic. The minimized makespan given by this heuristic is as follows.

Table 8: Solution of CDS heuristic with J4-J3-J2-J1 when **α**=0.6

Job	M1	M2	M3	M4	M5	M6
J4	0	22.16	22.16	54.42	76.66	97.87
J3	10.25	32.41	44.52	54.42	91.88	110.01
J1	32.45	56.7	56.7	56.7	108.23	125.96
J2	32.45	80.9	106.15	134.325	169.69	169.69

Again when the value of the risk factor is being increased, the makspan by the CDS heuristic also be increased for the uncertain processing time of the jobs. When **α**=0.6, then the total completion time is 169.69 seconds which is shown in the above mentioned tableau.

NEH Heuristic

Step 1: Find the total work content for each job using Expression $T_j = \sum_{i=1}^{m} P_{ij}$
Step 2: Arrange the jobs in a work content list according to decreasing values of T_j
Step 3: Select first two jobs from the list and from two partial sequences by inter changing the place of two jobs. Compute C_{max} the value of partial sequences. Of the two sequences, discard the sequence having larger value of C_{max}, Call the lower value of C_{max} as incumbent sequence.
Step 4: Pick the next job and put in incumbent sequence. Calculate value of C_{max} of all sequences
Step 5: If there is no job left in work content list to be added to Incumbent sequence, Stop go to step 4.

Sequence analysis by NEH Heuristic:

1st Case: For processing times when **α**=0.5

Step 1: Taking J2 & J4
Sequences: J2-J4 &J4-J2 and makespan 155.7875 & 134.375 respectively
Step 2: Choosing J4-J2 & taking J1
Sequences: J4-J2-J1 & J4-J1-J2 & J1-J4-J2 and makespan 166.7, 158.925 & 180.95
Step 3: Choosing J4-J1-J2 & taking J3
Sequences: J4-J1-J2-J3 & J4-J1-J3-J2 & J4-J3-J1-J2 & J3-J4-J1-J2 and makespan 186.1125, 158.8, 168.975 & 168.975 seconds.

Table 9: Solution of NEH heuristic with J4-J1-J3-J2 when **α**=0.5

JOB	M1	M2	M3	M4	M5	M6
J4	0	22.025	22.025	54.15	76.2875	97.4125
J1	22.125	46.25	46.25	54.15	92.5375	113.2875
J3	32.3	46.25	58.275	58.275	107.6625	125.35
J2	32.3	70.375	95.5	123.5625	158.8	158.8

2nd Case: For processing times when **α**=0.6
Step 1: Taking J2 & J4
Sequences: J2-J4 &J4-J2 and makespan 156.44 & 135.15 respectively
Step 2: Choosing J4-J2 & taking J1
Sequences:J4-J2-J1 & J4-J1-J2 & J1-J4-J2 and makespan 167.45, 159.4 & 181.6
Step 3: Choosing J4-J1-J2 & taking J3
Sequences: J4-J1-J2-J3 & J4-J1-J3-J2 & J4-J3-J1-J2 & J3-J4-J1-J2 and makespan 186.76, 159.4, 169.9 & 169.9 seconds.

Table 10: Solution of NEH heuristic with J4-J1-J3-J2 when **α**=0.6

JOB	M1	M2	M3	M4	M5	M6
J4	0	22.16	22.16	54.42	76.66	97.87
J1	22.2	46.45	46.45	54.42	93.01	113.82
J3	32.45	46.45	58.56	58.56	108.23	125.96
J2	32.45	70.65	95.9	124.075	159.44	159.44

6. Result Analysis

We have applied four optimization tool to determine the optimal make span where Palmer's and CDS heuristic has given a very high completion time to get the final product though these are better than traditional scheduling of the industry. And finally we have gotten the same but better, comparing to Palmer's and CDS heuristic,

Table 11: Comparison among Matrix Manipulation in MATLAB, Palmer, CDS and NEH Heuristics

No. of observation	Technique	Optimal sequence	Make span (When α=0.5)	Make span (When α=0.6)
01	Matrix Manipulation in MATLAB	J4-J1-J3-J2	158.8	159.4
02	Palmer's Heuristic	J2-J3-J1-J4	166.7	167.45
03	CDS Heuristic	J4-J2-J2-J1	168.825	169.69
04	NEH Heuristic	J4-J1-J3-J2	158.8	159.44

results for matrix manipulation in MATLAB and NEH algorithm whereas NEH algorithm takes much time to find out the makespan. By applying any of them we can get our optimum make span though.

7. Conclusion

In this paper it has been proposed that matrix manipulation method in MATLAB to solve flow shop scheduling problems for minimizing make span. The problem up to 4 jobs and 6 machines under uncertain processing times have been considered for comparative analysis among our proposed matrix manipulation method in MATLAB and Palmer's heuristic, CDS heuristic & NEH heuristic. From the analysis, it has been found that our used matrix manipulation method in MATLAB and NEH heuristic show the minimum value of make span time when compared to other heuristics. But NEH and other heuristics take long time for the calculation and it will be very difficult to solve if the size of the problem is very large. In this regard our proposed matrix manipulation method in MATLAB is able to solve any problem of any size such as n jobs and m machines without any manual calculation and gives the output at a very short time. Moreover if you wish to see the completion time required for any of the sequences of very large size problem then our proposed system will help you must and very firstly. But it is not at all possible in NEH or other heuristic because of they do not find all the job sequences and their respective make span. In our MATLAB coding, you will just have to put the processing times for every jobs for corresponding machines .The work can be extended by making a comparison of the effectiveness of the coding for the proposed matrix manipulation method with other coding.

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