

Modeling and Optimization of Aggregate Production Planning by Time-varying Acceleration Coefficients PSO

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Abstract

The aim of this paper is to apply Time-varying Acceleration Coefficients PSO (TVACPSO) for modeling and optimization of aggregate production planning problem. TVACPSO is a modified and updated form of Particle Swarm Optimization (PSO). Most practical decisions made to solve APP problems usually consider total costs; we have eliminated other objective functions of APP in this case. There was several variables problem with constraints which was solved in MATLAB. The proposed approach attempts to minimize total costs with reference to inventory levels, labor levels, and overtime, subcontracting and backordering levels. An industrial case is presented to demonstrate the feasibility of applying the proposed method to real APP problems. Consequently, the proposed TVACPSO approach yields an efficient APP compromise solution.

Keywords: Aggregate production planning (APP), Time-varying Acceleration Coefficients PSO (TVACPSO), cost minimization.

1. Introduction

Aggregate production planning is a process by which a company determines ideal levels of capacity, production, subcontracting, inventory, stock out & even pricing over a specified time horizon. It uses intermediate-range capacity planning that typically covers a time horizon (2 to 12 months), although some companies it may extend to as much as 18 months. The planning horizon is often divided into periods. For example, a one year planning horizon may be composed of six one-month periods plus two or three month periods. In other words, It is concerned with planning overall production of all products combined over a planning horizon for a given (forecast) demand schedule. The goal of aggregate is to achieve a production plan that will effectively utilize the organization's resources to satisfy expected demand. Planner must make decisions on output rates, employment levels and changes, inventory levels and changes, back orders, and subcontracting in or out. Aggregate planning might seek to influence demand as well as supply. If this is the case, variables such as price, advertising, and product mix might be used. If changes in demand are considered, then marketing, along with operations, will be intimately involved in aggregate planning. Aggregate planning is essentially a big-picture approach to planning. The purpose of aggregate production planning is to achieve a production plan that will effectively utilize the organizations resource to satisfy expected demand. There are many solving procedure for APP problem but in this paper we have used a new algorithm Time-varying Acceleration Coefficients PSO (TVACPSO). TVACPSO is a modified and updated form of Particle Swarm Optimization (PSO). The algorithm of PSO emulates from behavior of animals societies that don't have any leader in their group or swarm, such as bird flocking and fish schooling. PSO algorithm is a multi-agent parallel search technique which maintains a swarm of particles and each particle represents a potential solution in the swarm. All particles fly through a multidimensional search space where each particle is adjusting its position according to its own experience and that of neighbors. In a PSO method, all particles are initiated randomly and evaluated to compute fitness together with finding the personal best (best value of each particle) and global best (best value of particle in the entire swarm). In this study, TVACPSO is a relatively new approach for solving optimization problem is employed to solve the proposed APP problem due to its simplicity, speed, and robustness.

2. Literature Review

Aggregate production planning is the problem to determine the resource capacity needed to meet demand in the production line. Shorten product life cycle in market & fickle customer perceptions push the researchers to choose this broad area to research. Wang and Fang (2001) [1] modeled an APP with fuzzy variables and in this study the decision variables, the demands and subcontracting cost are accepted as triangular fuzzy numbers. Tang et al (2000) [2] formulated a multi-product APP problem as a fuzzy quadratic programming with both fuzzy demand and fuzzy constraint appears in the same model. A fuzzy multi-product APP problem was solved Fung et al. (2003) [3] and the APP problem has only one objective (minimizing the total cost) and only the demands and the capacities are defined as fuzzy. For solving the multi-product APP problem in a fuzzy environment a fuzzy multi-objective linear programming model was developed by Wang and Liang (2004) [4]. Wang and Liang (2005) [5] handled a fuzzy multi-product APP problem and is modeled as a multi-objective linear programming problem and all of the objectives are defined as fuzzy with imprecise aspiration levels. Ning et al. (2006) [6] established a multiproduct aggregate production planning (APP) decision making model in fuzzy random environments. Baykasoglu and Gocken (2006) [7] proposed a multi-objective tabu search (TS) based solution method to solve fuzzy goal programs. Tavakkoli-Moghaddam et al. (2007) [8] presented a fuzzy APP model for make-to-stock environments. Ling (2008) [9] developed a fuzzy multi objective linear programming (FMOLP) model with piecewise linear membership function to solve integrated multi product and multi time period production/ distribution planning decisions (PDPD) problems. Torabi et al (2009) [10] presented a fuzzy hierarchical production planning comprised of two decision-making levels. Chen and Huang (2010) [11] proposed a solution procedure that is able to find the fuzzy objective value of the fuzzy APP model. Based on ranking methods of fuzzy numbers and tabu search Baykasoglu and Gocken (2010) [12] proposed a direct solution method to solve fuzzy multi-objective aggregate production planning problem. Ramezani et al (2012) [13] presented a mixed integer linear programming (MILP) model for general two-phase aggregate production planning systems. Zhang et al (2012) [14] built a mixed integer linear programming (MILP) model to characterize mathematically the problem of APP with capacity expansion in a manufacturing system including multiple activity centers. Some meta-heuristic algorithms were also employed to solve APP problems. Raa et al (2013) [15] presented a mixed integer linear programming formulations for the planning problem and a metaheuristic solution approach based on the models. Chakraborty and Hasin (2013) [16] used three distinct scenarios simultaneously with different genetic algorithm options for solving multiple objectives of fuzzy linear programming models including escalating factors. Chen and Huang (2014) [17] investigated multiproduct, multi-period APP problems with several distinct types of fuzzy uncertainties. Particle Swarm Optimization (PSO) is another relatively new bio-inspired algorithm that may be used to find optimal or near-optimal solutions to numerical and quantitative problems. It was originally developed by a social psychologist, James Kennedy and Russel Eberhart (1995) [18]. El Mounayri et al. (2003) [19] used PSO to predict parameters of surface roughness in end milling. Prakasvudhisarn (2004) [20] used PSO to determine minimum tolerance zones of all basic form features for discrete parts inspection. Some researchers applied PSO to solve optimization problems with constraints. Hu et al. (2003) [21] modified the PSO to solve constrained nonlinear problems by preserving only feasible solutions. In 2002, A. Salman [22] published a paper on the use of particle swarm optimization for task assignment problem. Again H. J. Escalante et al [23] worked on particle swarm model design. In 2007, M. F. Tasgetiren et al [24] researched on the use of PSO algorithm for make span and total flow time minimization in flow shop sequencing problem.

3. Algorithm

3.1. Time-varying Acceleration Coefficients PSO

PSO has multiple particles, and every particle consists of its current objective value, its position, its velocity, its personal best value, that is the best objective value the particle ever experienced, and its personal best position, that is the position at which the personal best value has been found. Time-varying Acceleration Coefficients PSO (TVACPSO) does not only change the inertia weight w , but also the acceleration coefficients, i.e., the personal c_1 and global best weight c_2 , over time. The idea is to have a high diversity for early iterations and a high convergence for late iterations. TVACPSO uses the following iteration to determine the velocities:

- Step1: Assume the number of particles.
- Step2: Initialize the position of the particles.
- Step3: Evaluate the objective function for each particle.
- Step4: Set the initial velocities of each particle.

Step5: Find the personal best and global best position

Step6: New velocities calculation for each particle by using the following equation,

$$v^{(i)}(n+1) = w_{(n)}v^{(i)}(n) + c_1r_1^{(i)}(n)[x_p^{(i)}(n) - x^{(i)}(n)] + c_2r_2^{(i)}(n)[x_g(n) - x^{(i)}(n)],$$

$$n = 0, 1, 2, \dots, N-1,$$

Where the personal best weight c_1 and the global best weight c_2 at every iteration n are calculated using the following equations:

$$w_{(n)} = w_s - (w_s - w_e) \frac{n}{N}$$

$$c_1(n) = c_{1s} - (c_{1s} - c_{1e}) \frac{n}{N}$$

$$c_2(n) = c_{2s} - (c_{2s} - c_{2e}) \frac{n}{N}$$

where $c_1(n)$ is the personal best weight at iteration n , $c_2(n)$ is the global best weight at iteration n , c_{1s} is the personal best weight designated for the first iteration, c_{1e} is the personal best weight designated for the last iteration N , c_{2s} is the global best weight designated for the first iteration, and c_{2e} is the global best weight designated for the last iteration N .

Step7: Find updates position for each particle by using the following equation

$$x^{(i)}(n+1) = x^{(i)}(n) + v^{(i)}(n+1), n = 0, 1, 2, \dots, N-1,$$

Step8: Continue this process from step 3 to step 8.

4. Problem Formulation

4.1. Problem Description & Notation

The following notations were employed for formulating the APP problem:

N	Types of products
T	Planning horizon
z	Total production cost
D_{nt}	Forecasted demand for n th product in period t (units)
a_{nt}	Regular time production cost per unit for the n th product in period t (Tk./unit)
Q_{nt}	Regular time production for n th period in period t (units)
b_{nt}	Overtime production cost per unit for the n th product in period t (Tk./unit)
O_{nt}	Overtime production for n th period in period t (units)
c_{nt}	Subcontracting cost per unit for the n th product in period t (Tk./unit)
S_{nt}	Subcontracting volume for n th period in period t (units)
d_{nt}	Inventory carrying cost per unit for the n th product in period t (Tk./unit)
I_{nt}	Inventory level in period t for n th product (units)
e_{nt}	Backorder cost per unit for the n th product in period t (Tk./unit)
B_{nt}	Backorder level for n th period in period t (units)
g_t	Regular-time wages in period t (Tk./man-day)
W_t	Workforce level in period t (man-days)
k_t	Cost to hire one worker in period t (Tk./man-days)
H_t	The number of workers hired in period t (man-days)
m_t	Cost to layoff one worker in period t (Tk./man-days)
F_t	The number of worker laid off in period t (man-days)
i_{nt}	Hours of labor per unit of n th product in period t (man-hour/unit)
r_{nt}	Hours of machine usage per unit of n th product in period t (machine-hour/unit)
δ	Regular time per worker (man-hour/man-day)
α_t	The fraction of regular-time workforce available for use in overtime in period t

- V_{nt} Warehouse spaces per unit of nth product in period t (ft²/unit)
- W_{tmax} Maximum labor level available in period t (man-days)
- M_{tmax} Maximum machine capacity available in period t (machine-hours)
- V_{tmax} Maximum warehouse available in period t (ft²)

4.2. Objective Function

$$\text{Min } z = \sum_{n=1}^N \sum_{t=1}^T (a_{nt} Q_{nt} + b_{nt} O_{nt} + c_{nt} S_{nt} + d_{nt} I_{nt} + e_{nt} B_{nt}) + \sum_{t=1}^T (k_t H_t + m_t F_t + g_t W_t)$$

4.3. Constraints

Constraint on carrying inventory,

$$I_{n(t-1)} - B_{n(t-1)} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} = D_{nt} \quad \text{for } \forall n, \forall t$$

Constraint on labor level,

$$W_t = W_{t-1} + H_t - F_t \quad \text{for } \forall t$$

$$\sum_{n=1}^N i_{nt} Q_{nt} \leq \delta W_t \quad \text{for } \forall t$$

$$\sum_{n=1}^N i_{nt} O_{nt} \leq \alpha_t \delta W_t \quad \text{for } \forall t$$

$$W_t \leq W_{tmax} \quad \text{for } \forall t$$

Constraint on Machine capacity and Warehouse space,

$$\sum_{n=1}^N r_{nt} (O_{nt} + Q_{nt}) \leq M_{tmax} \quad \text{for } \forall t$$

$$\sum_{n=1}^N V_{nt} I_{nt} \leq V_{tmax} \quad \text{for } \forall t$$

Non-negativity constraints on decision variables,

$$Q_{nt}, O_{nt}, S_{nt}, I_{nt}, B_{nt}, W_t, H_t, F_t \geq 0$$

5. Case Description

In this study required data was taken from CCKL, one the leading company in RMG sector in Bangladesh. Tables 1 and 2 summarize the forecast demand, related operating cost, and capacity data. Other relevant data are as follows:

1. Initially there is no backorder ($B_{n0}=0$) and initial inventory (I_{n0}) of product 1 is 500 units and product 2 is 200 units.
2. Regular time per worker (δ) is 8 hours. Beginning workforce level (W_0) is 1,000 man-days.
3. The cost associated with regular, hiring and layoffs are Tk.195, Tk.205, Tk.115, per worker per day, respectively.
4. Setup cost for period 1 is 220 and period 2 is 210.
5. Hours of labor per unit of any periods are fixed to 0.033 man-hours for product 1 and 0.05 man hours for product 2. Hours of machine usage per unit for each of the two planning periods is 0.1 machine hours for product 1 and 0.08 machine hours for product 2. Warehouse spaces required per unit are 1 square feet for product 1 and 1.5 square feet for product 2. Setup time for product 1 is 0.6 machine-hours and product 2 is 0.5 machine-hours.
6. Fraction of regular workforce level available for overtime for both periods is 0.4.

Table1: Forecasted demand, maximum labor and machine and warehouse capacity data

Item (Units)	Periods	
	1	2
D_{1t} (pieces)	1400-1500	3000-3100
D_{2t} (pieces)	1600-1700	800-900
W_{tmax}	225	225
M_{tmax}	400	500
V_{tmax}	1000	1000

Table2: Related operating cost data

Product	a _{nt} (Tk./Unit)	b _{nt} (Tk./Unit)	c _{nt} (Tk./Unit)	d _{nt} (Tk./Unit)	e _{nt} (Tk./Unit)
1	22	40	27	3.5	42
2	20	40	30	4	47

6. Results and Findings

The APP decision problem presented here was solved by Time Varying Acceleration Coefficient PSO (TVACPSO). It was run in MATLAB 2010a by a PC with configuration Intel (R) Core(TM) i3-2310M CPU @ 2.10 GHz, 2 GB RAM. We have used $w_s = 0.9$, $w_e = 0.4$, $c_{1s} = 2.5$, $c_{2s} = 0.5$, $c_{1e} = 0.5$ and $c_{2e} = 2.5$. The optimal value when applying TVACPSO to minimize the total costs was **Tk. 2, 82, 249**. In this problem TVACPSO excelled in all criterion. It had the better objective function value in shortest time. In this case, only one objective function which is cost related is considered. But in practice, the firms may be operating under competing criteria and the primary objective may not necessarily be only minimizing cost. Finally, the TVACPSO approach is useful for solving APP decision problems and can generate better decisions within very short time.

7. Conclusion & Future Works

The results show that TVACPSO is much more effective in this type of case. Besides, fewer parameters selection has made it much easier to work with TVACPSO. In future we can work with total uncertain condition. Besides some modifications regarding velocity upgrade or velocity clamping can be made which will result in even faster performance. And obviously there's an option that we can perform some comparative study of TVACPSO with standard PSO and different modified form of PSO. Also TVACPSO may compare with other optimization algorithms and find suitable options for solving engineering optimization problems.

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