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A Production Inventory Model for Different Classes of Demands with Constant Production Rate Considering the Product's Shelf-Life Finite

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Abstract

This paper unfolds how a model is developed on the basis of market demands and company's production pattern. It advances in quest of optimum cost considering the product's shelf-life finite. The paper discusses about a production inventory model where company produces items with a constant rate, but demands vary due to the customers' needs. Without having any sort of backlogs, production starts. Reaching at the desired level of inventories, it stops production. After that due to demands along with the deterioration of the items it initiates its depletion and after certain periods the inventory gets zero. It is assumed that the decay of the products is level dependent. The objective of this paper is to find out the optimum cost and time.

Key words:Production inventory, Shelf-life time, Demand class, Production rate.

1. Introduction

In minimizing inventory cost this paper develops an Inventory Model of Deterministic Demand of materials which have the definite shelf-life. The Model advances by considering the constant production rate, small amount of decay, varying demand pattern, while reaching in certain amount of inventory level the production stops. Harries [1] presented the famous economic order quantity (EOQ) formulae. Whitin [2] was the first researcher who develops the inventory model with decay for fashion goods. Ghare and Schrader [3] first pointed out the effect of decay and discovered EOQ model. Rosenblatt and Lee [4] assumed the time as important factor for stock. Jamal, Sarker and Mondal [5] used single and Ekramol [6,7] used various production stages. Abdullah and Chaudhuri [8] and Jia-Tzer and Lie-Fern [9] included the defective items with imperfect process and backorders in the model. Teng, Chern and Yang [10] considered fluctuating demand and Skouri and Papachristos [11] discussed continuous review model. Vinod [12] described time dependent deteriorating items and Brojeswar, Shib and Chaudhuri [13] discussed with rework able items and supply distribution. The important formulae from Naddor [14] and Whitin [15] has been introduced to develop this paper. Subsequently, the model is formulated by proving that the total variable is convex, which shows that the optimum inventory cost is minimal.

2. Assumptions

a. Production rate is constant which starts when inventory is zero and decay is vary small.
b. Inventory level is highest at $T = 3t$. From this point, the old items must be delivered early to avoid its decay. Since, the production stops while inventory is highest, inventory depletes quickly due to demand.

3. Notations

λ = Production rate and a_i = demand rate at time $T=(i-1)xt$ to ixt, where i= 1 to 3.

μ = Very small amount of constant decay rate for unit inventory. After the production stops μ is 'zero'.

$I(\theta)$ = Inventory level at instant θ and I_4 = average inventory while no production occurs in the lemma.

$Q=0, Q_1, Q_2$ and Q_3 which depicts the inventory level respectively at $T=0, t, 2t$ and $3t$.

$Q(T)$ = Inventory at time T, Q_n = inventory considering the demand pattern index n and m = An integer.

x = Demand size during time t, n = demand pattern index, K_0 = Set up cost and h = average holding cost.

q = Inventory level while production stops and S = inventory after production stops at the beginning.

dT = Small portion of T, $W=a=$ demand, while no production occurs and $V=\theta=$ small time segment.

$TC(Q_1)$ = Total cost in terms of Q_1 and Q_1^* = optimum order quantity.

4. Development of the model

At the beginning, while time $T=0$, the production starts with zero inventory with the rate λ which remains constant for entire production cycle. But demands will vary time to time which is shown in the figure 1.

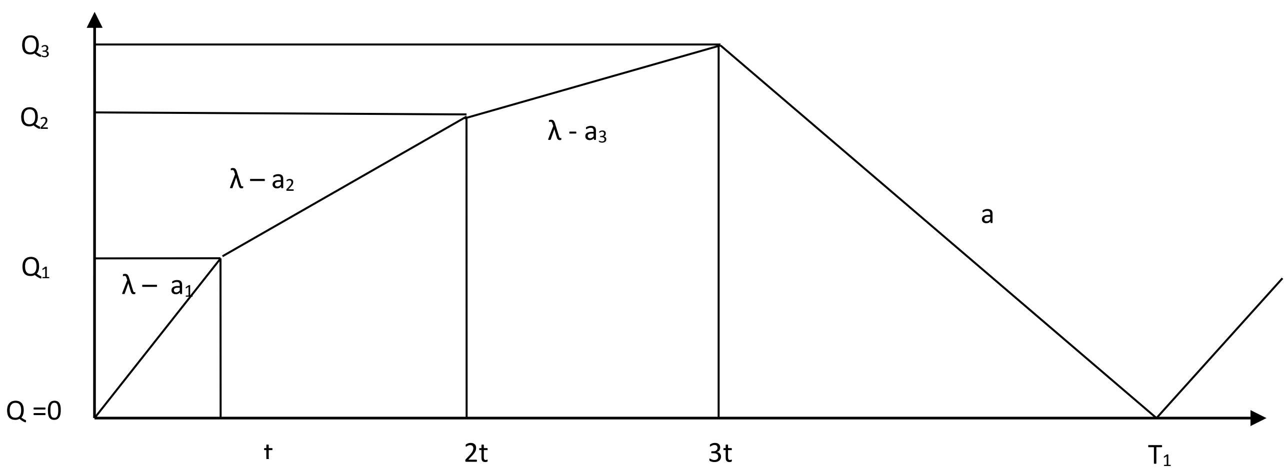

Fig. 1. Inventory level at various stages

From the above figure we get the value of t in different time segments and those are,

$$t=\frac{Q_1}{\lambda-a_1}=\frac{Q_2-Q_1}{\lambda-a_2}=\frac{Q_3-Q_2}{\lambda-a_3} \tag{1}$$

Therefore we get,

$$Q_2=Q_1+\frac{Q_1(\lambda-a_2)}{\lambda-a_1} \tag{2}$$

$$Q_3=Q_1+\frac{Q_1(\lambda-a_2)}{\lambda-a_1}+\frac{Q_1(\lambda-a_3)}{\lambda-a_1} \tag{3}$$

During $T=0$ to t, inventory increases at the rate of $\lambda-a_1-\mu I(\theta)$ and we get the differential equation as:

$$\frac{d}{d\theta}I(\theta)+\mu I(\theta)=\lambda-a_1$$

Applying the boundary condition $\theta=0$ and $I(\theta)=0$ we get the solution as,

$$I(\theta)=\frac{\lambda-a_1}{\mu}(1-e^{-\mu\theta})$$

Considering up to second degree of μ and using equation (1), total un-decayed inventory during $\theta=0$ to t,

$$I_1=\int_0^t I(\theta)d\theta=\left[\frac{\lambda-a_1}{\mu}(\theta+\frac{e^{-\mu\theta}}{\mu})\right]_0^t=\frac{Q_1{}^2}{2(\lambda-a_1)} \quad (4)$$

During $T=t$ to $2t$, applying the boundary condition at $\theta=t$, we consider, $I(\theta)=Q_1$.Then we get,

$$I(\theta)=\frac{\lambda-a_2}{\mu}+(Q_1-\frac{\lambda-a_2}{\mu})e^{\mu t}e^{-\mu\theta} \quad (5)$$

Being μ as a small quantity neglecting its higher power, the total un-decayed inventory during $\theta=t$ to $2t$ is,

$$I_2=\int_t^{2t} I(\theta)d\theta=t(\frac{\lambda-a_2}{\mu})+(Q_1-\frac{\lambda-a_2}{\mu})(t-\frac{\mu t^2}{2})$$

$$=\frac{Q_1{}^2(3\lambda-2a_1-a_2)}{2(\lambda-a_1)^2} \quad (6)$$

During $T=2t$ to $3t$, similar approach and boundary condition at $\theta=2t$, $I(\theta)=Q_2$ (say) being used, and μ being very small neglecting its higher power, we get the total un-decayed inventory as,

$$I_3=(\frac{Q_1}{\lambda\text{-}a_1})(\frac{\lambda-a_3}{\mu})+\{\frac{Q_1}{\lambda\text{-}a_1}-\frac{\mu Q_1{}^2}{2(\lambda-a_1)^2}\}\{Q_1+\frac{Q_1(\lambda-a_2)}{\lambda-a_1}-\frac{\lambda-a_3}{\mu}\}$$

$$=\frac{Q_1{}^2(5\lambda^2-7\lambda a_1-2\lambda a_2-\lambda a_3+2a_1{}^2+2a_1a_2+a_1a_3-Q_1\mu a_1}{2(\lambda-a_1)^3} \quad (7)$$

After reaching the desired level of inventories, the production stops and in this stage inventory reaches zero due to constant demand and negligible amount of decay.

Lemma: If the maximum inventory level is Q_3 and demand occurs in a uniform way, the amount of inventory will be as,

$$I_4=\frac{3Q_1\lambda-Q_1a_1-Q_1a_2-Q_1a_3}{2(\lambda-a_1)} \quad (8)$$

Proof: We know from Naddor [14], that the demand pattern can be generally represented as

$$Q(T)=S-x\sqrt[n]{T/t} \quad (9)$$

And on the basis of above equation from Naddor [9], we can express the average amount of inventory by

$$I_4 = \frac{q}{2} + \frac{1}{t} m \int_0^V (Q_n - Q_1) dT \tag{10}$$

where, $Q_n = W - W \sqrt[n]{T/V}$ which can be compared with equation no (9). In our case we use the last time segment $T_1 - 3t$, the demand occurs uniformly and as we neglected decay i.e. $\mu \to 0$ and comparing as $I_1 \to I_4$, $q \to Q_3$, $t \to T_1 - 3t$, $V \to \theta$, $T \to t$, $W \to a$, $Q_3 \to ma$, i.e. $m \to Q_3/a$, and demand pattern index $n = 1$.

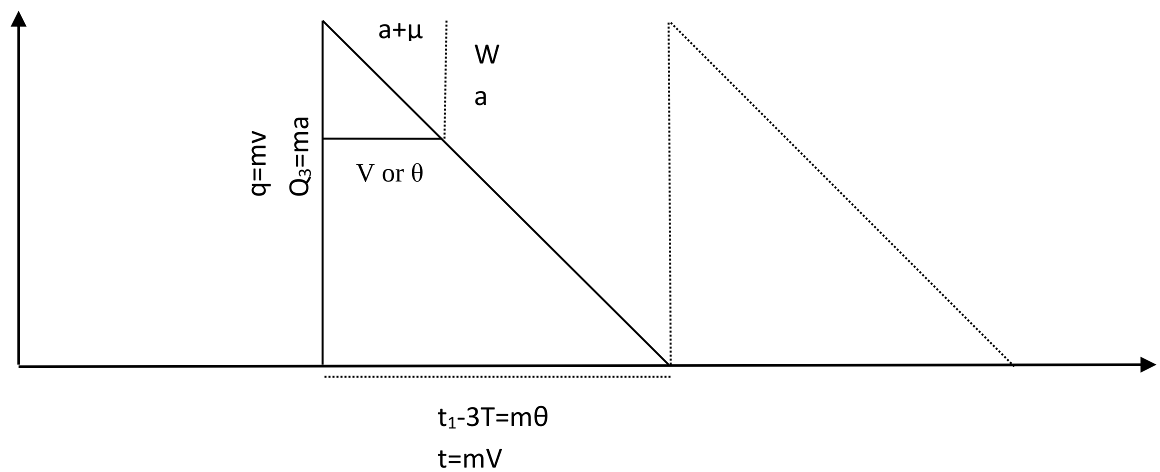

Fig. 2. Inventory level while no production occurs

Putting these values in the equation no (10), we get the following result,

$$I_4 = \frac{Q_3}{2} = \frac{3Q_1\lambda - Q_1a_1 - Q_1a_2 - Q_1a_3}{2(\lambda - a_1)} \tag{11}$$

Hence, from the equation number (8) and (11) we get the proof.

Total time cycle

Now, total time cycle can be expressed as,

$$T_1 = 3t + (T_1 - 3t) = 3t + m\theta = \frac{3Q_1}{\lambda - a_1} + \frac{Q_3}{a}\left(\frac{a}{a+\mu}\right) = \frac{Q_1(3\lambda + 2a - a_2 - a_3)}{a(\lambda - a_1)} \tag{12}$$

Total cost function

We use the equation no (6), (8), (9)and (10) to get the total cost, $TC(Q_1) = \frac{K_0 + h(I_1 + I_2 + I_3 + I_4)}{T_1})$

$$TC(Q_1) = \frac{K_0 a(\lambda - a_1)}{Q_1(3\lambda + 2a - a_2 - a_3)} + \frac{ha(\lambda - a_1)}{Q_1(3\lambda + 2a - a_2 - a_3)}\left\{\frac{Q_1^2}{2(\lambda - a_1)} + \frac{Q_1^2(3\lambda - 2a_1 - a_2)}{2(\lambda - a_1)^2} + \right.$$

$$\frac{Q_1^{\,2}(5\lambda^2 - 7\lambda a_1 - 2\lambda a_2 - \lambda a_3 + 2a_1^{\,2} + 2a_1 a_2 + a_1 a_3 - Q_1 \mu a_1}{2(\lambda - a_1)^3} + \frac{3Q_1\lambda - Q_1 a_1 - Q_1 a_2 - Q_1 a_3}{2(\lambda - a_1)}\}$$

$$= \frac{K_0 a(\lambda - a_1)}{Q_1(3\lambda + 2a - a_2 - a_3)} + \frac{haQ_1(9\lambda^2 - 14\lambda a_1 - 3\lambda a_2 - \lambda a_3 + a_1^{\,2} + a_1 a_2 + a_1 a_3)}{2(\lambda - a_1)^2(3\lambda + 2a - a_2 - a_3)} \qquad (13)$$

To determine the optimum order quantity Q_1^* and to verify that the equation no (13) is convex in Q_1, we must show that the first and second derivative of the equation (13) with respect to Q_1^* is zero and positive respectively. Hence, the convex properties imply that the first derivative $\frac{d}{dQ_1}TC(Q_1) = 0$ and the second derivative,

$$\frac{d^2}{dQ_1^{\,2}}TC(Q_1) = \frac{2K_0 a(\lambda - a_1)}{Q_1^{\,3}(3\lambda + 2a - a_2 - a_3)} = \frac{2K_0 a(\lambda - a_1)}{Q_1^{\,3}\{(\lambda - a_2) + (\lambda - a_3) + (\lambda + 2a)\}}$$

which is always positive as the quantity $K_0, a, Q_1, \lambda - a_1, \lambda - a_2, \lambda - a_3$ are positive.

Therefore, total cost is convex in Q_1. Hence, for optimum value of Q_1 the total cost function will be minimum and by Hadley and Whitin [15], we get the optimum order quantity Q_1^* as below,

$$Q_1^* = \sqrt{\frac{2K_0 a(\lambda - a_1)^3}{ha(9\lambda^2 - 14\lambda a_1 - 3\lambda a_2 - \lambda a_3 + a_1^{\,2} + a_1 a_2 + a_1 a_3)}} \qquad (14)$$

5. Numerical illustration with sensitivity analysis

Let, the parameters, $K_0 = 100, h = 2, a_1 = 1, a_2 = 2, a_3 = 3, a = 2, \lambda = 6, \mu = 0.01$. Then from (13) and (14) we get the optimum order quantity Q_1^* = 8.07 units and total optimum cost $TC(Q_1^*)$ = 14.58 units. Total cost decreases if demand a_1, a and λ increases; total cost increases if the demand $a_2$and a_3 increases.

Order quantity (Q_1) verses total cost (TC)

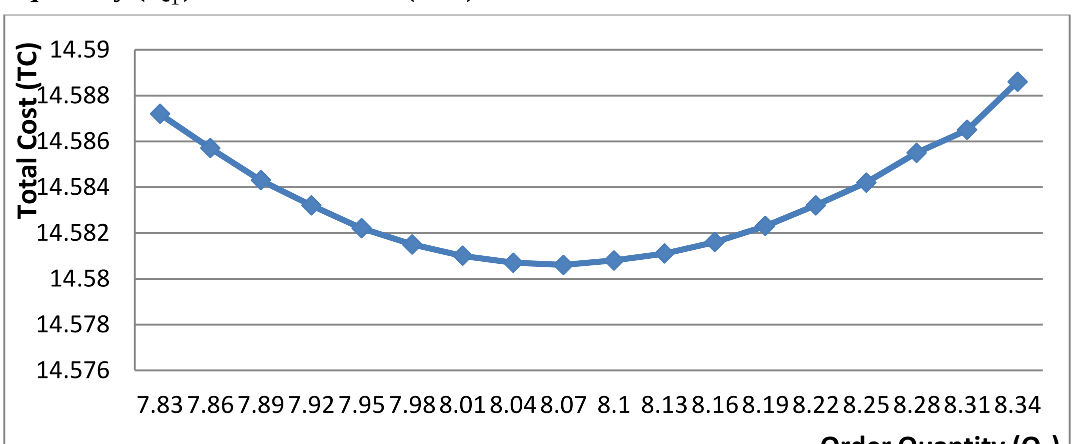

Fig. 3. Quantity Verses Total Cost

6. Conclusion

Total cost decreases in the first stage as it has sufficient inventory with respect to its demand increases. This cost increases during the second and third stages due to its reduced inventory level as the production stops at the end of this stages having increasing demand. Total cost increases in the fourth stage, while demand increases after no production. The Model could establish that with a particular order level (i.e. Q_1^* = 8.07 units) total cost is minimum (i.e. TC = 14.58 units). Before and after this point Total Cost increases sharply. This paper discussed that, in a varying demand pattern how a model is developed by receiving appropriate amount of order considering the market demands, product's shelf-life and company's production rate, which has ultimately ensured the optimum inventory cost.

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