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Inventory system at service-facility with N-policy consider reneging and rejection of pool customers

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Abstract

In this paper, we consider (S,s)inventory system. We assume that customer arrive to the system according a Poisson process with parameter $\lambda\rangle 0$. When inventory level depletes to s due to demands or service to a buffer customer, an order for replenishment is taken placed. The lead time is exponentially distributed with parameter γ .Any demand that takes place when the pool is full and inventory level is zero, is assumed to be lost forever. During the time of waiting in the pool customers can get impatience and leave the system with rate $\theta(1-\beta)$can stay in the pool with rate $\theta\beta$ Due to some reasons like to fail to show the proper documents, customer can get the rejection from the server during the time of taking the service with rate $\mu(1-\delta)$ or can get service with rate $\mu\delta$. The server will be on mode when the number of pool customer is $\geq N$, otherwise it is off mode. When inventory level is zero then arrival customer may enter the pool with $\theta(1-\beta)$ rate $\lambda\alpha$ and with rate will lost forever. Customers directly go to the pool (waiting room) that has finite capacity $M < \infty$.When number of customers reach to N , server becomes on mode to provide the service provided that the items are available in the stock. When number of waiting customers reach to $N-1$ again server becomes idle Customers are served as the basis of FIFS. In the present model, it is considered that during the time of waiting, customers can get impatient and can leave the system. Moreover the server has the authorization to reject the customers due to lack of eligibility or fail to show the proper documents to get services. The steady state probabilities are determined, some system characteristics are derived, numerical illustrationsare provided and sensitivity analysis is also made.

Key words: Reneging, Rejection, pooled customer and Inventory.

Introduction:

In inventory models the major objective consists of minimizing the total inventory cost and to balance the economics of large orders or large production runs against the cost of holding inventory and the cost of going short. Staring from a simple lot size formula a huge amount of research is done in inventory modeling. At the earlier research inventory control was treated as a separate identity from the service providing system. But the dimension is changed over the last three decades. Our model is on continuous-review inventory system with lost sales of customer that arrives during stock out. The lost sales situation arises in many cases, where the intense competition allows customer to choose another service point. This can be considered as a typical situation for being described by pure inventory Model. Lost sale are usually known as losses of customers. There is a huge amount of literature on loss system, especially in connection with telegraphic and communication system, where losses usually occur due to limited server capacity. But there is another occurrence of loses due to balking or reneging of impatient customers. Inventory system with service facilities was firstly considered by Sigman and simchi Levi[1]. Inthatpaer they considered M\G\1 queue with limited inventory system. An approximation procedures are use to find performance descriptions models, in which the interaction of queuing for service and inventory control in integrated. In a sequence of papers Berman and his coworkers [2-6] investigated the behavior of service systems with related to inventory system. Schwarz et al[7] characterized the earlier approaches in the following manner: They defined a Markovian system process and then used classical optimization methods to find the optimal control strategy of the inventory system. All those models assumed that the demand which arrives during time the inventory is zero is backlogged. The model varied with respect to lead-time distribution, the service time distribution, waiting room size, order size and reorder policy.

Islam and his co-workers [9-10] built some inventory models related to postponed demand, reneging pool customers and rejection of customers from the system in service facilities. In this paper we introduce Inventory system at service facility with N-policy. Consider reneging and rejection of pool customers. Customers direct go to the pool region and get service. The strategy of our investigation in this paper is as follows; we start the observation of Islam etal that paper the inventory system is started when the inventory levels is When the number of customers reach to N server becomes on mode to provide the service provided that the items are available in the stock. When number of waiting customers reach to $N-1$ again the server becomes idle. From the pool customers can get impatience and can leave the system with rate $\theta(1-\beta)$ and can stay in the pool with rate $\theta\beta$ Due to the same reasons like to fail to show the proper documents customer can get the rejection from the server.

Assumption:

1. Initially the inventory level is S
2. Interarrival times of demands are exponentially distributed with parameter λ .
3. Lead time is exponentially distributed with parameter γ .
4. Maximum pool capacity is M.
5. Demand that arrives when the pool is full, demand will be lost forever.
6. During the time of waiting in the pool; customers can get impatience and can leave the system with rate $\theta(1-\beta)$ can stay in the pool with rate $\theta\beta$.
7. Due to some reasons like to fail to show the proper documents, customers can get the rejection from the server at service epoch with rate $\mu(1-\delta)$ or can get service with rate $\mu\delta$.
8. The server will be on mode for the pool size if $j = N, N+1, \ldots\ldots, M$ and in off mode if $j \le N-1$.
9. When inventory level is zero then arrival customer may enter the pool with rate $\lambda\alpha$ and with rate $\lambda(1-\alpha)$ will lost forever.

Notation:

$I(t)$ = Inventory level at time t . ; λ = Arrival rate of customers to the system. γ = Lead time parameter. ; $N(t)$ = Number of customers in the pool. $X(t)$ = Represents the server'smode. ; μ = service rate of the system.

$\psi = [\mu(1-\delta)+\theta(1-\beta)]$; $\phi = [\lambda+\mu(1-\delta)+\theta(1-\beta)]$;

$\xi = [\lambda\alpha+\mu(1-\delta)+\theta(1-\beta)]$; $\eta = [\lambda+\mu+\theta(1-\beta)]$

Model and Analysis;

In this model, the inventory system is stared when the inventory levels is S and the system is in OFF mode. Demands follows Poisson Process with rate λ Inventory level will be deplete due to items provided to the customers from the pool customers can get impatience and can leave the system with rate $\theta(1-\beta)$ can stay in the pool with rate $\theta\beta$ and Due to some reasons like to fail to show the proper documents, customers can get the rejection from the server during the time of taking the service with rate $\mu(1-\delta)$ or can get service with rate $\mu\delta$ .When inventory level reaches in the state $(-N+1,0)$ the very next demand converts the system on mode from off mode i.e., $(-N,1)$. In that stage, if further demand arrives that will be lost forever. The inventory level $I(t)$ take the values in the set $A = \{-N, -N+1, \ldots\ldots, 0, \ldots\ldots, S\}$ By our assumptions {I(t), t≥0} does not follow Markov Process. To get a two dimensional Markov process, we incorporate the process {δ(t), t≥0 } into {I(t), t≥0} process where δ(t) is defined by,

Now, {I(t), δ(t), t≥0}is a two dimensional continuous Markov Process defined on the state space $E = (E_1 \cup E_2)$ where, $E_1 = \{(i,0) : i = -N+1, -N+2, \ldots, S\}$ and

$E_2 = \{(i,1) : i = -N, -N+1, \ldots, S-1\}$

Where $X(t) = \begin{cases} 0, & when\, server\, is\, off\, \bmod e \quad j \ge N, N+1 \ldots.. M \\ 1, & when\, server\, is\, on\, \bmod e \quad j \le N-1 \end{cases}$

$\{I(t), N(t), X(t);\ t \ge 0\}$ is a three dimensional Markov process with state space

$E_1 = \{0,1,2,3, \ldots\ldots\ldots\ldots S\};\ E_2 = \{0,1,2,3, \ldots\ldots\ldots\ldots N, N+1, \ldots\ldots, M\};\ E_3 = \{0,1\};\ E = E_1 \times E_2 \times E_3$

The infinitesimal generator can be obtained using the following arguments.

A) Arrival of customer makes a transition from
$(i, j, k) \rightarrow (l = i, m = j+1, n = 0)$ $i = 0.....S,\ J = 0,......N-2,\quad ,k = 0\quad l = i,\quad m = j+1$

B) A demand can convert the system off mode to on mode makes a transition
$(i, j, k) \rightarrow (l = i, m = j+1, n = 1)$ $i = 0.....S,\ j = N-1,\quad ,k = 0$

C) A customer demand can satisfied convert the system from on mode to off mode
$(i, j, k) \rightarrow (l = i, m = N, N-1,.....0, n = 0)$ $i = 0.....S,\ J = N........M,\quad ,k = 1$

D) Due to the replenishment inventory , the system makes a transitions
$(i, j, k) \rightarrow (l = i+Q, m = j, n = k)$ $i = 0.....S,\ j = N......M,\quad ,k = 1$

E) Due to the inventory, the system makes transitions.
$(i, j, k) \rightarrow (l = i+Q, m = j, n = k = 0)$ $i = 0.....S,\ J = 0......N-1,\quad ,k = 0$

On Mode:

γ $i = 0,1,2,..........S,\ l = i+Q$
j=N,..........M, m=j
K=1………, n=k

$\mu\delta$ $i = 1,2,........S\quad l = i-1$
$j = N+1..........M\quad m = j-1$
$k = 1\quad n = k = 1$

$\mu\delta$ $i = 1,2,........S\quad l = i-1$
$j = N\quad m = j-1$
$k = 1\quad n = k = 0$

$\mu(1-\delta)$ $i = 0,2,.........S\quad l = i$
$j = N+1......M\quad m = j-1$
$k = 1\quad n = k$

$\mu(1-\delta)$ $i = 0,2,........S\quad l = i$
$j = N\quad m = j-1$
$k = 1\quad n = k = 0$

$\theta(1-\beta)$ $i = 1,2,........S\quad l = i$
$j = N+1........M\quad m = j-1$
$k = 1\quad n = k$

$\theta(1-\beta)$ $i = 0,1,.......S\quad l = i$
$j = N\quad m = j-1$
$k = 1\quad n = k = 0$

λ $i = 1,..........S\quad l = i$
$j = N,...\text{M-1}\quad m = j+1$
$k = 1\quad n = k$

$\lambda\alpha$ $i = 0\quad l = i$
$j = N,...\text{M-1}\quad m = j+1$
$k = 1\quad n = k$

$-\gamma-\lambda-\theta(1-\beta)-\mu$ $i = 1,...,s\quad l = i$
$j = N,...M,\ m = j$
$k = 1,\quad n = k$

$-\lambda-\theta(1-\beta)-\mu$ $i = s+1,...S\quad l = i$
$j = N,...M-1,\ m = j$
$k = 1,\quad n = k$

$-\gamma-\lambda\alpha-\theta(1-\beta)-\mu(1-\delta)$ $i = 0\quad l = i$
$j = N,...M-1,\ m = j$
$k = 1,\quad n = k$

$-\gamma-\theta(1-\beta)-\mu(1-\delta)$ $i = 0\quad l = i$
$j = M\quad m = j$
$k = 1\quad n = k$

$-\gamma-\theta(1-\beta)-\mu(1-\delta)$ $i = 1,...,s\quad l = i$
$j = M\quad m = j$
$k = 1\quad n = k$

$-\theta(1-\beta)-\mu(1-\delta)$ $i = s+1,...,S\quad l = i$
$j = M\quad m = j$
$k = 1\quad n = k$

$-\gamma-\theta(1-\beta)-\mu$ $i = 1,2,...S,\ l = i$
$j = M,\quad m = j$
$k = 1,\quad n = k$

Off Mode:

γ $i = 0,1,................s\quad l = i+Q$
$j = 0,1..........\text{N-1}\quad m = j$
$k = 0\quad n = k = 0$

λ $i = 1,................S\quad l = i$
$j = 0,1..........\text{N-2}\quad m = j+1$
$k = 0\quad n = k = 0$

λ $i = 1,................S\quad l = i$
$j = \text{N-1}\quad m = j+1$
$k = 0\quad n = k = 1$

$\theta(1-\beta)$ $i = 0,1,................S\quad l = i$
$j = 0,1..........\text{N-1}\quad m = j-1$

$\lambda\alpha$ $i = 0\quad l = i$
$j = 0,1..........\text{N-2}\quad m = j+1$
$k = 0\quad n = k = 0$

$-\gamma-\lambda-\theta(1-\beta)$ $i = 1,...,s\quad l = i$
$j = 0,...,N\quad m = j$
$k = 0\quad n = k$

$-\gamma-\lambda\alpha-\theta(1-\beta)$ $i = 0\quad l = i$
$j = 0,...,N-1\quad m = j$
$k = 0\quad n = k$

$-\lambda-\theta(1-\beta)$ $i = \text{s}+1,...,\text{S}\quad l = i$
$j = 0,...,N-1\quad m = j$
$k = 0\quad n = k$

$k=0$ $\qquad$ $n=k=0$ |

Let us assumed $I(0)=S$ and $X(0)=0$

consider the transition probabilities: $P^{(S,0,0(i,j,k))}(t)=p\{I(t),N(t),X(t)=(i,j,0)\}|I(0),N(0),X(0)=(S,0,0)\}$

From now onwards we can write $P^{(i,j,k)(t)}$ for $P^{(S,0,0(i,j,k)(t))}$. The kolmogorve forward difference differential equation are given below.

When the system is OFF mode

$P^{(i,0,0)}=-\lambda P^{(i,0,0)}+\gamma P^{(i-Q,0,0)}+\psi P^{(i,1,0)}$;i = S..............s + 1 , j = 0

$P^{(i,j,0)}=-\lambda P^{(i,j,0)}+\psi P^{(i,j+1,0)}$;i = s..............1 , j = 0

$P^{(i,j,0)}=-\lambda\alpha P^{(i,j,0)}+\psi P^{(i,j+1,0)}$;i = 0 , j = 0

$P^{(i,j,0)}=\lambda P^{(i,j-i,0)}+\gamma P^{(i-Q,j,0)}+\psi P^{(i,j+1,0)}-\phi P^{(i,j,0)}$;i = S......s + 1 , j = 1

$P^{(i,j,0)}=\mu\delta P^{(i+1,j+1,1)}+\lambda P^{(i,j-i,0)}+\gamma P^{(i-Q,j,0)}+\psi P^{(i,j+1,0)}-\phi P^{(i,j,0)}$;i = s.............1, j = 1...........N - 1

$P^{(i,j,0)}=\lambda P^{(i,j-i,0)}+\gamma P^{(i-Q,j,0)}+\psi P^{(i,j+1,0)}-\phi P^{(i,j,0)}$;i = s.............1, j = 1...........N - 1

$P^{(i,j,0)}=\lambda\alpha P^{(i,j-i,0)}+\psi P^{(i,j+1,0)}-\xi P^{(i,j,0)}$;i = 0 , j = 1...........N

When the system is on mode

$P^{(i,j,1)}=\lambda P^{(i,j-i,1)}+\gamma P^{(i-Q,j,1)}+\psi P^{(i,m-1,1)}-\eta P^{(i,j,1)}$;i = S......s + 1 , j = 2........M - 1

$P^{(i,j,1)}=\lambda P^{(i,j-i,1)}+\gamma P^{(i-Q,j,1)}+\psi P^{(i,m-1,1)}-\eta P^{(i,j,1)}+\mu\delta P^{(i+1,M-i,1)}$;i = S - 1......s + 1 , j = 2........M - 1

$P^{(i,j,1)}=\lambda P^{(i,j-i,1)}+\gamma P^{(i-Q,j,1)}+\psi P^{(i,m,1)}-\eta P^{(i,j,1)}$;i = S , j = M

$P^{(i,j,1)}=\lambda P^{(i,j-i,1)}+\gamma P^{(i-Q,j,1)}+\psi P^{(i,m-1,1)}-\eta P^{(i,j,1)}$;i = S......s + 1 , j = 2........M - 1

$P^{(i,j,1)}=\lambda P^{(i,j-i,1)}+\gamma P^{(i-Q,j,1)}+\psi P^{(i,m,1)}-\eta P^{(i,j,1)}+\mu\delta P^{(i+1,M,1)}$; i = S - 1......s + 1 , j = 3........M - 1

$P^{(i,j,1)}=\lambda P^{(i,j-i,1)}+\gamma P^{(i-Q,j,1)}-\eta P^{(i,j,1)}$; i = S , j = M

$P^{(i,j,1)}=\lambda P^{(i,j-1,1)}+\psi P^{(i,j+1,1)}-\eta P^{(i,j,1)}+\mu\delta P^{(i+1,j+1,1)}$;i = s.....1 , j = N

$P^{(i,j,1)}=\lambda P^{(i,j-1,1)}+\psi P^{(i,j,1)}-\eta P^{(i,j,1)}+\mu\delta P^{(i+1,j,1)}$;i = s............1 , j = M

$P^{(i,j,1)}=\lambda\alpha P^{(i,j-1,1)}+\psi P^{(i,j+1,1)}-\xi P^{(i,j,1)}$;i = 0 , j = $N+1$...........M - 1

$P^{(i,j,1)}=\lambda\alpha P^{(i,j-1,1)}-\xi P^{(i,j,1)}$; i = 0 , j = M

Limiting Distribution:

The steady state probabilities for $(i,j)\in E$ of the system size are obtained by taking the limit at $t\rightarrow\infty$ on both sides of the above equations and solving them respectively .Note that under steady state condition

$\lim_{t\rightarrow\infty} P_{i,j,0}=0$; $\lim_{t\rightarrow\infty} P_{i,j,1}=0$

And $\lim_{t\rightarrow\infty} P_{i,j,0}=q_{i,j,0}$; $\lim_{t\rightarrow\infty} P_{i,j,1}=q_{i,j,1}$

The limiting distribution exists and satisfies the normalize condition

$$\sum_{i=-1}^{S}\sum_{j=0}^{N-1}P_{i,j,0}+\sum_{i=1}^{S}\sum_{j=N}^{M}P_{i,j,1}=1$$

The balancing equation can be written as

When the is in off mode:

$$q^{(i,0,0)} = \frac{\gamma}{\lambda} q^{(i-Q,0,0)} + \psi/\lambda \, q^{(i,1,0)} \quad ; i = S.............s+1 \; , j = 0$$

$$q^{(i,\,j,0)} = \frac{\psi}{\lambda} q^{(i,\,j+1,0)} \quad ; i = s.............1 \; , j = 0 \qquad q^{(i,\,j,0)} = \frac{\psi}{\lambda\alpha} q^{(i,\,j+1,0)} \quad ; i = 0 \; , j = 0$$

$$q^{(i,\,j,0)} = \frac{\lambda}{\phi} q^{(i,\,j-i,0)} + \frac{\gamma}{\phi} q^{(i-Q,\,j,0)} + \frac{\psi}{\phi} q^{(i,\,j+1,0)} \quad ; i = S......s+1 \; , j = 1$$

$$q^{(i,\,j,0)} = \frac{\lambda}{\phi} q^{(i,\,j-1,0)} + \frac{\mu\delta}{\phi} q^{(i+1,\,j+1,1)} + \frac{\psi}{\phi} q^{(i,\,j+1,1)} \quad ; i = s......1 \; , j = 1........N-1$$

$$q^{(i,\,j,0)} = \frac{\lambda}{\phi} q^{(i,\,j-1,0)} + \frac{\psi}{\phi} q^{(i,\,j+1,1)} \quad ; i = s......1 \; ; j = 1........N-1$$

$$q^{(i,\,j,0)} = \frac{\lambda\alpha}{\xi} q^{(i,\,j-1,0)} + \frac{\psi}{\xi} q^{(i,\,j+1,1)} \quad ; i = s......1 \; , j = 1........N-1$$

When the syatem is on mode

$$q^{(i,\,j,1)} = \frac{\lambda}{\eta} q^{(i,\,j-i,1)} + \frac{\gamma}{\eta} q^{(i-Q,\,j,1)} + \frac{\psi}{\eta} q^{(i,m-1,1)} \quad ; i = S......s+1 \; , j = 2........M-1$$

$$q^{(i,\,j,1)} = \frac{\lambda}{\eta} q^{(i,\,j-i,1)} + \frac{\gamma}{\eta} q^{(i-Q,\,j,1)} + \frac{\mu\delta}{\eta} q^{(i+1,\,M-1,1)} + \frac{\psi}{\eta} q^{(i,m-1,1)} \quad ; i = S-1......s+1 \; , j = 2........M-1$$

$$q^{(i,\,j,1)} = \frac{\lambda}{\eta} q^{(i,\,j-i,1)} + \frac{\gamma}{\eta} q^{(i-Q,\,j,1)} + \frac{X}{\eta} q^{(i,m-1,1)} \quad ; i = S \; , j = M$$

$$q^{(i,\,j,1)} = \frac{\lambda}{\eta} q^{(i,\,j-i,1)} + \frac{\gamma}{\eta} q^{(i-Q,\,j,1)} + \frac{\mu\delta}{\eta} q^{(i+1,\,M,1)} + \frac{\psi}{\eta} q^{(i,m,1)} \quad ; i = S-1......s+1 \; , j = 3........M-1$$

$$q^{(i,\,j,1)} = \frac{\lambda}{\eta} q^{(i,\,j-i,1)} + \frac{\gamma}{\eta} q^{(i-Q,\,j,1)} \quad ; i = S \; , j = M$$

$$q^{(i,\,j,1)} = \frac{\lambda}{\eta} q^{(i,\,j-i,1)} + \frac{\mu\delta}{\eta} q^{(i+1,\,j+1,1)} + \frac{\psi}{\eta} q^{(i,\,j+1,1)} \quad ; i = s......1 \; , j = N$$

$$q^{(i,\,j,1)} = \frac{\lambda}{\eta} q^{(i,\,j-i,1)} + \frac{\mu\delta}{\eta} q^{(i+1,\,j,1)} + \frac{\psi}{\eta} q^{(i,\,j,1)} \quad ; i = s......1 \; , j = M$$

$$q^{(i,\,j,1)} = \frac{\lambda}{\eta} q^{(i,\,j-i,1)} \quad ; i = s......1 \; , j = M \qquad q^{(i,\,j,1)} = \frac{\lambda\alpha}{\xi} q^{(i,\,j-i,1)} + \frac{\psi}{\xi} q^{(i,\,j+1,1)} \quad ; i = 0 \; , j = N+1........M-1$$

$$q^{(i,\,j,1)} = \frac{\lambda\alpha}{\xi} q^{(i,\,j-i,1)} \quad ; i = 0 \; , j = M$$

System Performance Measures

(a) Mean Inventory holds in the system

$$\alpha_1 = \sum_{i=1}^{S} \sum_{j=0}^{N-1} iP_{i,\,j,0} + \sum_{i=1}^{S} \sum_{j=N}^{M} iP_{i,\,j,1}$$

b) Expected number of customers reneging to the system

$$\alpha_2 = \sum_{i=0}^{S} \sum_{j=1}^{N-1} i\theta(1-\beta)P_{i,\,j,0} + \sum_{i=0}^{S} \sum_{j=N}^{M} i\theta(1-\beta)P_{i,\,j,1}$$

c) Expected number of customers rejection to the system:

$$\alpha_3 = \sum_{i=0}^{S} \sum_{j=N}^{M} \mu(1-\delta)P_{i,\,j,1}$$

d) The probability that a demand will be satisfied just after its arrival is,

$$\alpha_4 = \sum_{i=1}^{S} \sum_{j=0}^{N-1} P_{i,\,j0} + \sum_{i=1}^{S} \sum_{j=N}^{M} P_{i,\,J,1}$$

Steady State Cost Analysis of the mode:

Let us consider the costs under steady state as given below: -

L = the initial set-up cost of the system. C_1 = inventory carrying cost per unit per unit time.

C_2= Cost due to reneging per unit time. $C_{3=}$Cost due to rejection per unit time.
C_4 = Cost due to customer lost to the system So, the expected total cost to the system is,

$$E(TC) = L + C_1\alpha_1 + C_2\alpha_2 + C_3\alpha_3 + C_4\alpha_4$$

Numerical Illustrations:

The results we have obtained in steady state case may be illustrated through the following numerical example: Let Then we can get the following system performances by using the given set of parameters which is shown in table-1. $S = 5$, $N = 2$, $\lambda = 4$, $\delta = 0.8$, $\mu = 5$, , $L = 100$, $C_1 = 6$, $C_2 = 4$, $C_3 = 3$ and $C_4 = 2$.Then we can get the following system performance by using the given set of parameters which is shown in table-1.

Table 1: Different system performances by using a given set of parameters

Mean inventory holds in the system	0.72398146
Expected number of reneging customers in the system	0.1227514428
Expected number of rejectionscustomers in the system	0.11349607
Average customers lost to the system	0.11349607
Total cost to the system	105.175368

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Appendix

$\prod^{(5,0,0)}$	0.00267620	$\prod^{(4,0,0)}$	0.0100089	$\prod^{(3,0,0)}$	0.06417534	$\prod^{(2,0,0)}$	0.0024546	$\prod^{(1,0,0)}$	0.0012776	$\prod^{(0,0,0)}$	0.0001088
$\prod^{(5,1,0)}$	0.0066905	$\prod^{(4,1,0)}$	0.0250224	$\prod^{(3,1,0)}$	0.1666008	$\prod^{(2,1,0)}$	0.0153415	$\prod^{(1,1,0)}$	0.00798605	$\prod^{(0,1,0)}$	0.0023128
$\prod^{(5,2,1)}$	0.01672626	$\prod^{(4,2,1)}$	0.00494931	$\prod^{(3,2,1)}$	0.00404775	$\prod^{(2,2,1)}$	0.00211607	$\prod^{(1,2,1)}$	0.02004473	$\prod^{(0,2,1)}$	0.00051321
$\prod^{(5.3.1)}$	0.0836320	$\prod^{(4,3,1)}$	0.0091659	$\prod^{(3,1,1)}$	0.00467090	$\prod^{(2,3,1)}$	0.00676811	$\prod^{(1,3,1)}$	0.00698716	$\prod^{(0,3,1)}$	0.0289502
$\prod^{(5,4,1)}$	0.4599589	$\prod^{(4,4,1)}$	0.0036663	$\prod^{(3,4,1)}$	0.0033364	$\prod^{(2,4,1)}$	0.00233443	$\prod^{(1,4,1)}$	0.001791	$\prod^{(0,4,1)}$	0.3534654