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Production Inventory System with Different Rates of Production Considering Poisson Demand Arrivals

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Abstract

This paper represents a single product stochastic production inventory model with two different rates of production where demand arrives according to Poisson process. The model is built up based on matrix approach. In this current model, backlogs are permitted and assumed that the production will start from the inventory level where there are some predetermined backlogs. It is also assumed that the production rate is higher during the backlogs situation than the situation where there is no backlog. Some measures of the system performance in the steady state case are derived, some numerical illustrations are given and sensitivity analyses are provided.

Key words: Backlogs, Production Inventory, Stochastic Inventory system.

1. Introduction

During the last 40 years there has been a rapid growth of interest in scientific inventory control. Scientific inventory control is generally understood to be the use of mathematical model to obtain rules for operating inventory system. The subject has attracted such a wide interest that today every serious student in Mathematics. Management science and industrial Engineering areas are expected to have some experienced working with inventory Model. Originally, the development of Inventory models had practical application as an immediate objective to the large extent this is still true, but as the subject becomes older, better developed and more thoroughly explored, an increasing number of individuals are working with inventory models because they present interesting theoretical problems in Mathematics. For such individuals, practical application is not a major objective; although there is the possibility that their theoretical work may be helpful in practice at some future time.

2. Background of the Study

Many researchers have considered inventory model with the finite and infinite production rates. Dave and Choudhuri [5] considered finite rate of production. Bhonia and Maiti [1] have examined two models, in one model they considered production rate as a function of the on hand inventory and in another as a function of demand rate. Chaho-Ton Su. Change-Wang Lin & Chih-Hung Tsai [3] A deterministic Production Inventory model for deteriorating items with an exponential Declining Demand. Rein Nobel and Headen [13] have considered production inventory model with two discrete production modes. Perumal and Arivarignan [14] have considered a deterministic inventory model with two different production rates. A Krishnamoorthy andMohammad Ekramol Islam[7] considered (s,S) inventory system with postponed demands. Mohammad Ekramol Islam et.al [8] considered stochastic inventory system with different rates of production where backlogs were permitted. They also consider in that paper shelf-life of the inventoried items are infinity. Mohammad Ekramol Islam et.al [9] farther extended the result for perishable inventoried items.

In that paper switching time also considered and that has taken as a random phenomena. In both of their papers, they built up the models by using the concept of Kolmogorov differtial-difference equations. B. Sivakumar and G. Arivarignan [2] have considered an inventory system with postponed demands. Mohammad Ekramol Islam, K M Safiqul Islam and Shahansha Khan[10] have considered Inventory system with postponed demands considering reneging pool customers. Mohammad Ekramol Islam and Shahansha khan [11] have considered a perishable inventory model at service facilities for systems with postponed demands. Mohammad Ekramol Islam and Shahansha khan [12] further considered a perishable inventory systems with postponed demands considering reneging pool customers

In this paper, we have considered a stochastic production inventory system with two different rates of production with the possible slippage of production rate from one rate to another rate over time. For the stage (-N to 0) we have considered production rate μ_1 and that of the stage (0 to S) μ_2, where $\mu_1>\mu_2$. Such a situation is desirable since our production starts from a backlog situation and the model is built up by matrix approach.

3. Notations

$\lambda \to$ Arrival rate

$\mu_1 \to$ Production rate when inventory levels vary from –N to 0.

$\mu_2 \to$ Production rate when inventory levels vary from 0 to S.

N $\to$ Pre-determined backlogs quantity.

S $\to$ Maximum Inventory level.

I(t) $\to$ Inventory level at time t.

4. Assumptions

(1) Two different rates of production μ_1 and μ_2 are considered where $\mu_1 > \mu_2$.

(2) The process will continue producing at the rate μ_1 until the backlog vanishes and then will start producing at the rate μ_2 and will continue producing at the same rate up to order level (S) following exponential distribution.

(3) Production process will start when the inventory level reaches –N unit of items as a backlogs.

(4) When inventory level will reach the order level, production will be switched off

5. Methodology

In this model, the inventory system starts with a backlog and reaches in off mode at the inventory level S. Demand arrives at a rate λ following Poisson process. Inventory level will be depleted when customers demand will be satisfied. When inventory level reaches in the state $(-N+1,0)$ the very next demand converts the system on mode from off mode i.e, $(-N,1)$. In that stage, if further demand arrives that will be lost forever. The inventory level I (t) take the values in the set $A=\{-N,-N+1,\ldots\ldots,0,\ldots\ldots,S\}$

By our assumptions {I(t), t≥0} does not follow Markov Process. To get a two dimensional Markov process, we incorporate the process {δ(t), t≥0 } into {I(t), t≥0} process where δ(t) is defined by,

$$\delta(t)=\begin{cases}1 & \text{When production process is ON}\\ 0 & \text{Otherwise}\end{cases}$$

Now, {I(t), δ(t), t≥0}is a two dimensional continuous Markov Chain defined on the state space $E=(E_1 \cup E_2)$ where, $E_1=\{(i,0): i=-N+1,-N+2,\ldots,S\}$ and $E_2=\{(i,1): i=-N,-N+1,\ldots,S-1\}$

The infinitesimal generator matrix of the process $\tilde{A}=(a(i,j:k,l);(i,j),(k,l)\in E)$

$$\tilde{A} = \begin{array}{r} S,0 \\ S-1,0 \\ S-2,0 \\ .. \\ 1,0 \\ 0,0 \\ .. \\ -N+1,0 \\ -N,1 \\ -N+1,1 \\ .. \\ 0,1 \\ 1,1 \\ .. \\ S-2,1 \\ S-1,1 \end{array} \left[\begin{array}{cccccccccccccccc}
-\lambda & \lambda & 0 & .. & 0 & 0 & .. & 0 & 0 & 0 & .. & 0 & 0 & .. & 0 & 0 \\
0 & -\lambda & \lambda & .. & 0 & 0 & .. & 0 & 0 & 0 & .. & 0 & 0 & .. & 0 & 0 \\
0 & 0 & -\lambda & .. & 0 & 0 & .. & 0 & 0 & 0 & .. & 0 & 0 & .. & 0 & 0 \\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\
0 & 0 & 0 & .. & -\lambda & \lambda & .. & 0 & 0 & 0 & .. & 0 & 0 & .. & 0 & 0 \\
0 & 0 & 0 & .. & 0 & -\lambda & .. & 0 & 0 & 0 & .. & 0 & 0 & .. & 0 & 0 \\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\
0 & 0 & 0 & .. & 0 & 0 & .. & -\lambda & \lambda & 0 & .. & 0 & 0 & .. & 0 & 0 \\
0 & 0 & 0 & .. & 0 & 0 & .. & 0 & -\lambda-\mu_1 & \mu_1 & .. & 0 & 0 & .. & 0 & 0 \\
0 & 0 & 0 & .. & 0 & 0 & .. & 0 & \lambda & -\lambda & .. & 0 & 0 & .. & 0 & 0 \\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\
0 & 0 & 0 & .. & 0 & 0 & .. & 0 & 0 & 0 & .. & \lambda-\mu_2 & \mu_2 & .. & 0 & 0 \\
0 & 0 & 0 & .. & 0 & 0 & .. & 0 & 0 & \lambda & .. & \lambda & \lambda-\mu_2 & .. & 0 & 0 \\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\
0 & 0 & 0 & .. & 0 & 0 & .. & 0 & 0 & 0 & .. & 0 & 0 & .. & \lambda-\mu_2 & \mu_2 \\
\mu_2 & 0 & 0 & .. & 0 & 0 & .. & 0 & 0 & 0 & .. & 0 & 0 & .. & \lambda & -\lambda-\mu_2
\end{array}\right]$$

The above matrix can be obtained using the following arguments;-

A. The arrival of demand makes a transition from
   $(i, j) \rightarrow (k = i-1, l = 0, j = 0)$ if $i = S, S-1.... -N+2.$
   $(i, j) \rightarrow (k = i-1, l = j = 1)$ if $i = S-1.... -N+1$
B. Production of an item makes a transition
   $(i, j) \rightarrow (k = i+1, l = j = 1)$if $i = -N...S-2$
C. A demand can convert the system from off mode to on mode and can make a transition from.
   $(i, j=0) \rightarrow (k = i-1, l = j = 1,$ if $i = -N+1$
D. One unit of production can convert the system from one mode to off mode and can make a transition
   $(i, j=1) \rightarrow (k = i+1, l = j = o);$ if $\mathrm{i} = \mathrm{S}+1$

Inventory level which represents negative sign indicates intangible items (i,e backlogs,)

6. Steady State Analysis

It can be seen from the structure of matrix $\breve{A}$ that the state space E is irreducible. Let the limiting distribution be denoted by $p_{i,j}$:

$$p_{i,j} = \underset{t\to\infty}{Lt} \Pr[I(t), \delta(t) = (i, j)], \ (k, l) \in E$$

The limiting distribution exists and satisfies the following equations-

$$p\tilde{A} = 0 \qquad \text{and} \qquad \sum_{\text{i=-N+1,0}}^{\text{S}} \sum_{\text{j}=-N,1}^{\text{S-1}} p_{i,j} = 1$$

The first equation of the above yields the following set of equations:-

In off Mode

(i) $-\lambda p_{S,0} + \mu_2 p_{S-1,1} = 0$

(ii) $-\lambda p_{i,0} + \lambda p_{i+1,0} = 0; i = S-1....,-N+1$

(iii) $-\mu_1 p_{-N,1} + \lambda p_{-N+1,0} + \lambda p_{-N+1,1} = 0$

In On Mode

(i) $-(\lambda + \mu_2) p_{S-1,1} + \mu_2 p_{S-2,1} = 0$

(ii) $-(\lambda + \mu_2) p_{i,1} + \lambda p_{i+1,1} + \mu_2 p_{i-1,1} = 0 \ ; i = S-2, S-3,...,1$

(iii) $-(\lambda+\mu_2)p_{0,1}+\lambda p_{1,1}+\mu_1 p_{-1,1}=0$

(iv) $-(\lambda+\mu_1)p_{i,1}+\lambda p_{i+1,1}+\mu_1 p_{i-1,1}=0\ ;i=-1,-2,\ldots,-N+1$

The solutions are as follows

When the system is in off mode

$$p_{i,0}=\frac{1}{\rho_2}p_{(S-1,1)};i=S,S-1,\ldots,-N+1$$

When the system is in on mode

$$p_{S-2,1}=(1+\rho_1)p_{(S-1,1)}$$
$$p_{S-i,1}=(1+\rho_2)p_{(S-(i-1),1)}-\rho_2 p_{(S-(i-2),1)}\ \ ;i\geq 3,4,\ldots.,S$$
$$p_{(S-i,1)}=(\tau+\rho_1)p_{(S-(i-1),1)}-\rho_1 p_{(S-(i-2),1)}\ \ ;i=S+1$$
$$p_{S-i,1}=(1+\rho_1)p_{(S-(i-1),1)}-\rho_1 p_{(S-(i-2),1)}\ ;i=S+2,\ldots.,S+N$$
$$where,\rho_1=\frac{\lambda}{\mu_1},\rho_2=\frac{\lambda}{\mu_2},\tau=\frac{\mu_2}{\mu_1}$$

Where, $p_{(S-1,1)}$ can be obtained by using the following normalizing condition i.e.,

$$\sum_{i=-N+1}^{S}p_{i,0}+\sum_{i=-N}^{S-1}p_{i,1}=1$$

7. System Performance Measures

(a) Mean Inventory holds in the system

Let α_1 denote the average inventory level in the steady state. Then $\alpha_1=\sum_{i=1}^{S}ip_{i,0}+\sum_{i=1}^{S-1}ip_{i,1}$

(b) Expected backlogs hold in the system:

Let α_2 be the expected backlogs in the system in steady state. Then α_2 can be defined as:

$$\alpha_2=\sum_{i=-N+1}^{-1}|i|p_{i,0}+\sum_{i=-N}^{-1}|i|p_{i,1}$$

(c) Average number of Customer's lost to the system:

Let α_3 is the average number of customers lost to the system. Then α_3 can be defined as:

$$\alpha_3=\lambda p_{-N,1}$$

(d) The probability that a demand will be satisfied just after its arrival is,

$$\alpha_4=\sum_{i=0}^{S}p_{i,0}+\sum_{i=1}^{S-1}p_{i,1}$$

(e) Expected waiting time of a customer

$$E(T)=\sum_{n=0}^{2}[\frac{N-n}{\lambda}+\frac{n+1}{\mu_1}]q_{(n,0)}+\sum_{n=0}^{3}\frac{n+1}{\mu_1}q_{(-n,1)}$$

8. Steady State Cost Analysis of the model

Let us consider costs under steady state as given below: -

L = the initial set-up cost of the system.

C_1 = inventory carrying cost per unit per unit time.

C_2 = Backlog cost per unit time.

C_3 = Cost due to customer lost to the system

So, the expected total cost to the system is,

$E(TC) = L + C_1\,\alpha_1 + C_2\,\alpha_2 + C_3\,\alpha_3$

$$= L + C_1\left(\sum_{i=1}^{S} i p_{i,0} + \sum_{i=1}^{S-1} i p_{i,1}\right) + C_2\left(\sum_{i=-N+1}^{-1} |i|\, p_{i,0} + \sum_{i=-N}^{-1} |i|\, p_{i,1}\right) + C_3\,\lambda p_{-N,1}$$

Since the computation of the $\Pi's$ are recursive, it is very difficult to show the convexity of the total expected costs. However, it may possible for many cases to demonstrate the computability of the result and to illustrate the existence of local optima when the cost function is treated as function of only two variable which obviously a restricted case and hence avoided. Of course, it may possible to explore some of the very important characteristics of the system and also possible to do sensitivity analysis of the system.

9. Results

The results we have obtained in steady state case may be illustrated through the following numerical example:

Steady state probabilities in the system for the parameters of S = 10, N = 3, $\lambda = 3$, $\mu_1 = 5$, $\mu_2 = 4$, L = 100, $C_1 = 6$, $C_2 = 4$ and $C_3 = 3$ are as follows:

Table 1: Different system performances by using a given set of parameters

Mean inventory holds in the system	2.38591
Expected backlogs holds in the system	0.48549
Average customers lost to the system	0.21051
Probability that a demand will be satisfied just after arrival	0.47661
Total cost to the system	116.88895

10. Sensitivity Analysis

Table 2: Production rate μ_1 Vs Total cost

μ_1 value	α_1	α_2	α_3	Total Cost
5	2.38591	0.48549	0.21051	116.88895
6	2.37760	0.49503	0.21909	116.90299
7	2.65555	0.32571	0.12054	117.59776
8	2.54998	0.37984	0.15588	117.28688
9	2.75436	0.24349	0.08040	117.74132
10	2.96740	0.10266	0.00375	118.22629

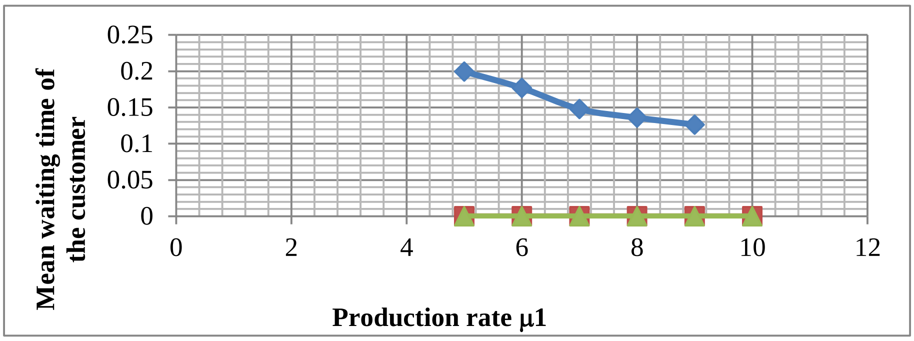

Fig. 1: Production rate Vs Waiting time

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