Transient Peak Reduction by Feedforward Control in Zero-compliance System with Displacement Cancellation Technique

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Abstract

A horizontal single-axis zero-compliance system using displacement cancellation technique is studied both analytically and experimentally. In the displacement cancellation technique, the zero-compliance characteristic is obtained by an infinite stiffness isolator connected with a positive stiffness in series. Previously, it was noticed that the transient responses of a zero-compliance system are unpleasant and do not match the objective function of a zero-compliance system. Since a feedforward control works parallel to the main control system without deteriorating the responses for other functions in the main controller, a feedforward (FF) control is added to the integral controller applied in the displacement cancellation technique. The responses with and without feedforward control are investigated and they are compared in this study. From experimental results, it is observed that the transient peaks of the experimental zero-compliance system are significantly reduced with feedforward control.

Keywords: Displacement cancellation, Infinite stiffness, Positive stiffness, Zero compliance.

1. Introduction

Precision vibration isolation plays an important role in acquiring the position accuracy in many advanced production and measurement industries. During the last three decades, the position and dimension accuracies in high-tech manufacturing processes have progressed to the submicron level [1]. Such rapid improvements in manufacturing processes have increased the importance of research on high-performance zero-compliance systems. A zero-compliance system possesses the characteristic of zero displacement to disturbance and is one of the effective approaches to obtain high precision by suppressing disturbances in high-tech manufacturing processes [2-3]. The integral control is the most usual approach to obtain zero-compliance to direct disturbance; however, a system with integral control is subjected to transient peak with comparatively longer settling time that causes the increasing of control cost. Moreover, conventional integral controlled systems are not suitable for disturbance cancellation transmitted directly from ground (base). In these regards, the authors have proposed an effective approach that is a series combination of positive and infinite stiffness isolators to obtain a zero-compliance system with the capability of suppressing ground disturbances as well [4].

There are two main types of disturbances: (i) direct disturbance (i.e., on-board disturbance); and (ii) ground disturbance (i.e., vibration transmitted from the ground to the isolation stage). The disturbances could be suppressed either by passive or active manner. The passive technique for suppressing disturbances could be employed by stiff spring for direct disturbance and soft stiffness spring for ground disturbance. However, a trade-off is inevitable between high and low stiffness isolator for suppressing the both direct and ground disturbances by a single vibration isolation system. The active vibration isolation is one of the effective means to overcome such dilemma [5]. In this article, the suppression of disturbances with active system is explicated. An active horizontal vibration isolation system equipped with eddy current gap sensors instead of accelerometer is developed in this study to reduce the overall cost.

In the previous study, the authors have implemented this concept (zero-compliance mechanism) to develop a vibration isolation system using negative stiffness technique [6]. In the negative stiffness technique, a positive stiffness isolator and a negative stiffness isolator of equal absolute stiffness are connected in series. A zero-power controlled magnetic suspension system was connected with a normal spring in series to isolate vibration because zero-power control magnetic suspension systems behave as if they have negative stiffness [6]. The characteristics of such system were further improved using a positive stiffness actuator instead of a normal spring [7]. In actual practice, however, the maintaining of same magnitude of stiffness of the isolators in the negative stiffness technique is difficult because negative stiffness control is very sensitive and easily deteriorates into nonlinear behavior that sometimes degrades the system from producing infinite stiffness. In contrast, the vibration isolation systems using the displacement cancellation technique can avoid this difficulty because the...
displacement cancellation technique is based on displacement rather than stiffness. In this study, the displacement cancellation that comprises integral control is employed to develop a horizontal zero-compliance system to suppress direct and ground disturbances.

Previously, the authors developed a zero-compliance system using displacement cancellation technique, and it was observed that a significant level of disturbance suppression occurred with the developed system [7]. Nevertheless, the transient displacements occurred for such zero-compliance system are often unpleasant and does not satisfy the objective function required to have for using in high-precision micro-manufacturing processes. The higher gain of the closed-loop system in designing the controller is one of the approaches to overcome this problem but practically it is problematic [8]. Therefore, in this study, feedforward (FF) control is added parallel with the integral control in the displacement technique to shorten the transient displacement as well as to improve the dynamics of the system. In a previous research, a feedforward control is added with negative stiffness control system used for vibration isolation [9]. In this research, feedforward control is added to the integral control system for disturbance cancellation where linear actuators (voice coil motor, VCM) are used as actuators. A new control network is considered as well so that the proposed controller does not hamper the other characteristic of the zero-compliance system developed in this study. The integral control system with feedforward control is connected in series with a positive stiffness system to obtain the characteristics of zero-compliance system. Theoretically and experimentally, it is shown that the feedforward control with proposed control approach can suppress the transient displacement sufficiently.

2. Displacement Cancellation Technique to Cancel Disturbances

In the displacement cancellation technique, one of the two series connected isolators has a soft positive stiffness (e.g., a soft coil spring) and the other is controlled to cancel displacement due to the soft positive stiffness isolator against a force, which is shown in Fig. 1. Because the stiffness \( k_1 \) is positive, the displacement of the upper table “ \( \Delta y \) ” takes place along the direction of disturbance. This displacement is cancelled by the upper isolator (Fig. 1), which is controlled with integral-proportional derivative (I-PD) control. Inherently, the upper isolator behaves as if it has a negative stiffness. The displacement \( \Delta y \) can be defined as follows:

\[
\Delta y = (y_1 + y_2) - (y_1 - \Delta y_1 + y_2 - \Delta y_2).
\]

where \( \Delta y_1 \) and \( \Delta y_2 \) are the displacement of the lower and upper isolators, respectively. The displacement \( \Delta y \) would be zero if the following condition is satisfied:

\[
0 = (y_1 + y_2) - (y_1 - \Delta y_1 + y_2 - \Delta y_2) \Rightarrow \Delta y_1 = -\Delta y_2.
\]

Equation (2) indicates that the static zero-compliance to a direct disturbance using the displacement cancellation technique occurs when the compression in one isolator is equal to the extension in the other isolator (i.e., \( |\Delta y_1| = |\Delta y_2| \)), as shown in Fig. 1. For a ground vibration, the combination of a middle mass and a soft isolator works as a mechanical filter that attenuates the transmitting of ground disturbances to the upper table. In addition, a low-pass filter in the feedback loop between the displacement of the middle mass and the actuator of the upper table can further improve the performance of ground disturbances transmissibility [7]. In this study, an electronic low-pass filter is inserted between the two tables of the developed experimental system using the displacement cancellation technique. The displacement-cancellation isolator and positive-stiffness isolator are created using VCMs guided by I-PD and PD control, respectively. A simple application of PID control hardly can achieve the criteria associated to a vibration isolation system (infinite stiffness and low stiffness). Therefore, an integral control with regard to a command signal and a PD control are applied in the same time to attain the desired infinite stiffness in the experimental system; this is known as I-PD control.

3. Basic model of system and controller design

![Fig. 1 Concept of displacement cancellation technique](image1)

![Fig. 2 Basic model of a control system suspended horizontally](image2)
To design the controller parameters, the isolation table is assumed to be guided by the actuator without any effect of internal interference. A basic single degree-of-freedom model with VCM for designing the controller is shown in Fig. 2. Here, the moving table with mass \( m \) is assumed to be moved along \( x \)-axis of horizontal translation motions. The motion equation of the table guided by a VCM is

\[
m \ddot{x} = f_a + f_d,
\]

where symbols \( m \), \( x \) denote mass, displacement of the isolation table respectively, and \( f_a, f_d \) denote actuator’s thrust force, direct disturbance acting on the table, respectively.

Thrust exerted by an actuator is proportional to coil current \( i \), so the force \( f_a \) can be expressed as

\[
f_a = k_i i,
\]

where \( k_i \) denotes actuator’s thrust force coefficient. From Eqs. (3) and (4), the transfer function representation of the actuator’s dynamics is given by

\[
X(s) = \frac{1}{s^2} (b_0 I(s) + d_0 F_d(s)) ,
\]

where each Laplace-transformed variable is denoted by its capital and \( b_0 = \frac{k_i}{m} \), \( d_0 = \frac{1}{m} \).

In displacement cancellation technique, the displacement due to direct disturbance is cancelled by inserting a command signal into the controller. Therefore, the actuator dynamic sown by Eq. (5) can be revised for displacement cancellation control as follows:

\[
X(s) = \frac{1}{s^2} \left[ \frac{k_i I(s)}{m} \right].
\]

Here I-PD control is employed to cancel the variation of displacement. If displacement cancellation is taken place due to the command signal \( r \), then the control current of the I-PD control can be represented as follows:

\[
i^{(d)}(s) = -P_i^{(d)} \int (x - r) dt - P_d^{(d)} x - P_v^{(d)} \dot{x}.
\]

The Laplace transform of Eq. (7) is written as follows:

\[
I^{(d)}(s) = \frac{P_i^{(d)}}{s} r(s) - \left( \frac{P_d^{(d)}}{s} + P_v^{(d)} s \right) X(s),
\]

where \( P_i^{(d)}, P_d^{(d)} \) and \( P_v^{(d)} \) are proportional, derivative and integral gains of the I-PD controller, respectively. Substituting of Eq. (8) into Eq. (6) leads to the transfer function presentation of the closed-looped system (Fig. 2) with I-PD control:

\[
\frac{X(s)}{R(s)} = \frac{k_i P_i^{(d)}}{m t_c^{(d)}(s)},
\]

where \( t_c^{(d)}(s) \) represents the characteristics equation of the controlled system and is expressed as follows:

\[
t_c^{(d)}(s) = s^3 + \left( \frac{k_i}{m} \right) P_i^{(d)} s^2 + \left( \frac{k_i}{m} \right) P_d^{(d)} s + \frac{k_i}{m} P_v^{(d)}.
\]

\[
t_d^{(s)}(s) = (s^2 + 2 \zeta_0 \omega_0 s + \omega_0^2)(s + \omega_0) = s^3 + \omega_1 s^2 + \alpha_1 s + \alpha_0.
\]

Equation (10) indicates that the system with I-PD control is a 3rd-order system. According to the pole assignment method, gains of the I-PD controller are determined uniquely by comparing characteristics equation

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Fig. 3 Basic feedback model with FF control

Fig. 4 Two series connected tables in experimental system
(10) with the ideal characteristic equation for a third order system (Eq. 11), and they are obtained as follows:

\[ p_i^{(d)} = \frac{ma_0}{k_1}, \quad p_d^{(d)} = \frac{ma_1}{k_1}, \quad p_s^{(d)} = \frac{ma_2}{k_1} \]

where symbols \( \omega_0 \) and \( \zeta_r \), denote angular frequency and damping ratio, respectively, and they are used to specify the closed-loop poles of 2nd-order system.

Similarly, the gains for PD controller are determined by comparing with an ideal 2nd-order system as a conventional PD control system is 2nd-order system. Finally the gains for PD controller are obtained as follows:

\[ p_d^{(p)} = \frac{ma_0}{k_1}, \quad p_v^{(p)} = \frac{ma_1}{k_1} \]

where \( P_d^{(p)} \) and \( P_v^{(p)} \) are proportional and derivative gains of the PD controller, respectively.

4. Displacement Cancellation Technique with Feedforward Control

Transfer function representation of the system regarding force \( F_d \) to displacement \( x \) without feedforward control and with feedforward control shown in Fig. 3 can be expressed by the Eq. (12) and (13), respectively.

\[
\frac{X(s)}{F_d(s)} = \frac{1}{ms^2 + k_i G(s)} = \frac{\text{Num}}{\text{Den}},
\]

\[
\frac{X(s)}{F_d(s)} = \frac{k_i H(s) + 1}{ms^2 + k_i G(s)} = \frac{\text{Num}}{\text{Den}},
\]

where \( H(s) \) denotes the feedforward gain. In Eq. (13), it is noticed that appropriate choosing of feedforward gain \( H(s) \) can make the numerator part to become zero; hence, it is possible to occur zero transient displacement with feedforward control.

To realize suppression of both direct and ground disturbances by a single system, the experimental zero-compliance system shown in Fig. 4 is fabricated based on the concept of series combination of two stages; the motion equations of the moving tables are as follows:

\[ m_0 s^2 \dot{X}_1(s) = k_1 I_1(s) - k_2 I_2(s), \]

\[ m_2 s^2 \dot{X}_2(s) = k_2 I_2(s) + F_d(s), \]

The control network of the displacement cancellation technique incorporated feedforward control that is applied to the experimental setup is shown in Fig. 5. Since unpleasant transient peak is one of the problems to obtain a disturbance isolation table, the control network (Fig. 5) is employed to shorten the transient peak in this study. The corresponding control current to the middle table and the isolation table are given, respectively, as follows:

\[
I_1(s) = -(P_{d1} + P_{v1}s)(X_1(s) - X_0(s)) + F_d H_1(s),
\]

\[
I_2(s) = \frac{P_v}{s} \{ (X_2(s) - X_1(s)) + G_F(s)(X_1(s) - X_0(s)) \} - \frac{F_d}{s}(X_2(s) - X_1(s)) + H_2(s).
\]

Here \( H_1(s) \) and \( H_2(s) \) define the feedforward gains for middle table and isolation table, respectively. Substituting currents shown by Eqs. (16) and (17) into Eqs. (14) and (15) yields the dynamics to disturbance of the middle table and the isolation table, respectively, as follows:

\[
\frac{X_1(s)}{F_d(s)} = \frac{1}{k_1(\dot{I}_1(s) - k_2 H_2(s))} \left[ k_1 H_1(s) - k_2 H_2(s) \right] \left[ m_2 s^2 + \dot{I}_2(s) + \frac{k_2 P_v}{s} \right],
\]

\[
\frac{X_2(s)}{F_d(s)} = \frac{1}{k_1(\dot{I}_1(s) - k_2 H_2(s))} \left[ k_1 H_1(s) - k_2 H_2(s) \right] \left[ \frac{k_2 P_v}{s} - (1 - G_F(s) + k_2 \dot{I}_2(s)) \right],
\]

where

\[
\dot{I}_1(s) = k_1(P_{d1}^{(p)} + P_{v1}^{(p)} s), \quad \dot{I}_2(s) = k_2 \left( P_d^{(d)} + P_v^{(d)} s \right), \quad \dot{I}_3(s) = k_2 \left( \frac{P_v^{(d)}}{s} \right),
\]

\[
\dot{I}_c(s) = (m_2 s^2 + \dot{I}_2(s) + \dot{I}_2(s)(1 - G_F(s))(m_2 s^2 + \dot{I}_2(s) + \dot{I}_3(s)) \quad - (\dot{I}_2(s) + \dot{I}_2(s))(\dot{I}_2(s) + \dot{I}_3(s)(1 - G_F(s))).
\]

It can be noted that the transient displacement would be disappeared if the numerator part of the transfer functions given in Eqs. (18) and (19) becomes zero. Thus to discard the transient displacement by feedforward...
control, the feedforward gains are selected so that the numerator of the Eqs. (18) and (19) becomes zero which is obtained while the gains are considered as follows:

\[ H(s) = -\frac{1}{k_1} G(s) = -\frac{1}{k_2}. \]  \hspace{1cm} (20)

Eq. (20) indicates that it is necessary to predict the disturbance on the table for the suppressing transient peak by feedforward control. Hence, it is recognized theoretically that the transient displacement could be discarded with feedforward control by choosing appropriate gain for the feedforward control and predicting the disturbances exactly. To confirm this theoretical finding, several experiments have been conducted with and without feedforward control. The experimental behaviors of the system with and without proposed feedforward control network (Fig. 5) are shown in the following section.

5. Experimental Results

Several experiments have been conducted to confirm the improvement of the dynamic behavior of the zero-compliance system utilizing displacement cancellation technique including feedforward control. Direct dynamic disturbance is applied on the isolation table and the respective displacements of the isolation table are measured. The magnitude of was kept constant for the both cases of with and without feedforward control in the displacement cancellation technique.

A certain step input in terms of disturbance on the isolation table is applied and corresponding step responses of the table are measures. The step responses of the isolation table with and without feedforward control against the same step input are measured and shown in Fig. 7. The infinite stiffness in the isolation table with respect to base in response to direct disturbance is obtained, inherently the isolation table returns to its initial position and maintain zero compliance in the steady state. Nevertheless, it is noticed that the transient peaks of the tables are appeared at the beginning of step up and step down of step disturbance. This phenomenon occurs because active feedback control may not accomplish sufficiently fast responses as delays are contained in the control scheme inherently. From theory, these unpleasant transient displacements can be minimized by cancelling the numerator part of the Eq. (19), and the cancellation of this numerator part is accomplished by using feedforward control.

The feedforward signals with the gains presented in Eq. (20) are simultaneously added to the I-PD and PD controllers in the displacement cancellation technique. It is noticed in the Fig. (7) that the transient peak is almost suppressed as well as settling time is reduced to a minimum level while the feedforward control is considered. However, a little displacement is still noticed with feedforward control, which is mainly due to the existing of nonlinearity like friction in the experimental system. Hence, the vibration isolation characteristics can further be improved, if feedforward control along with proper nonlinear compensator is added to the controller.

The frequency responses of the isolation table to direct disturbance are measured with and without feedforward control. The swept sinusoidal direct disturbance is applied on the isolation table and corresponding frequency responses of the table are measured and shown in Fig. (8). In frequency responses, the swept sinusoidal direct disturbance was the input signal and the corresponding table displacement (output) was measured over the frequency of swept sinusoidal direct disturbance. It is obvious from Fig. (8) that the frequency response of the isolation table is improved significantly with the feedforward control. The displacements of the tables are almost

![Fig. 5 Block diagram of displacement cancellation technique with feedforward control](image-url)
zero at a low-frequency region which satisfies that the experimental system possesses the characteristic of zero-compliance.

6. Conclusions

An active zero-compliance system was developed by using the displacement cancellation technique where an integral controlled VCM is connected with a positive stiffness system in series. The behaviors of the experimental system were investigated with and without feedforward control in response to disturbances. The experimental results showed that the experimental system with feedforward control, utilizing the control network proposed in this study, suppressed the transient peak of displacement significantly compared to the without feedforward control. This suppression of transient peak was validated theoretically in this study as well. In addition, the enhanced frequency responses of the experimental system were confirmed with feedforward control compared to the without feedforward control.

6. References