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An Analytical Study of Passive Anti-Roll Tanks

Gazi Md. Khalil[1], Syed Marzan-ul Hasan[1], Md. Ehsan Khaled[2]
[1]Dept. of Naval Architecture & Marine Engineering, BUET, Dhaka-1000
[2]Yokohama National University, Japan
E-mail: hasan_name06@yahoo.com

Abstract

This paper presents a detailed theoretical analysis using the principle of conservation of energy and spring-mass system analogy for the oscillation of water in two identical tanks connected by pipe. An expression is derived for the frequency of oscillation of water as a function of the affecting parameters in anti-roll tanks. It is mathematically demonstrated that the circular and rectangular tanks produce the same frequency of oscillation of water if their cross-sectional areas remain same.Furthermore, this paper takes into account the additional damping forces which arise from friction between the moving fluid and the surrounding tank wall. This makes the theoretical model more realistic. The computational results are plotted graphically and then physically interpreted in order to demonstrate the effect of the depth of water and the area of cross-section of each vertical tank; the effect of the length, diameter and number of connecting pipes; and the effect of damping on the oscillation of water in the tanks. The results of this analytical study are expected to be useful in the design of anti-rolling tanks which will effectively reduce the rolling motion of ships.

Keywords: Natural frequency of oscillation, rolling motion, anti-roll tank, passive control, dampingeffect

1. Introduction

The understanding and control of rolling motion i.e. the rotational motion about the longitudinal axis of a ship is essential for efficient ship manoeuvring, ensuring crew comfort and proper functioning of onboard equipment. Till date, a variety of roll control mechanisms like bilge keels, fin stabilizers, gyro stabilizers, anti-roll tanks, etc. have been subjected to comprehensive theoretical study and detailed practical experimentation.

Anti-roll tanks (ART) generate anti-rolling moments using differences in water heights at the two limbs of U-tank whose oscillation is exactly out of phase to the incoming wave excitations. The use of ART has an added advantage of controlling the rolling motion even if the vessel is not underway. An Active ART uses pumps or any mechanical means to generate the moments from fluid flow and often become impractical in case of large vessels. Passive anti-roll tank (PART), on the other hand, with no active means to generate counter moments relies solely on accurate tuning techniques to ambient conditions, and therefore, require a greater study across the linear and non-linear range.

Froude [11] was the first to use anti-roll tanks to reduce roll motion. He installed water chambers in the upper part of the ship. The free-surface effect of the water tank lengthened the period of the rolling motion but reduced the ship's stability; consequently, the system was abandoned. Frahm[4], a German naval engineer revived the method, and was the first to understand the importance of placing the horizontal leg or cross-duct in the U-tube. An active counterpart of Frahm's passive tank was conceived by Minorsky[8]. A restoring moment was developed by transferring the water directly with a proper phase from one leg of the U-tube tank to the other at a high rate. Chadwick [6]derived the governing equations for rolling motion in calm water and found that the axis of roll does not necessarily pass through the centre of gravity of the ship. The hydrodynamic effects are considered linear and hence superimposable. Crockett [9] in his report illustrated a step-by-step preliminary design procedure for designing, installing, and tuning of passive anti-roll tanks based on some empirical formulae and practical experiences.Bell and Walker[5] foundthat the positioning of the tanks is a critical issue and the typical designs and characteristics of vessels must be taken into account while designing an effective ART. Gawad*et al*. [1] paid attention to the fluid motion inside the tank itself. They mentioned that proper tuning of the anti-roll tank to the ship's natural frequency is very important in reducing the roll motion. Dallinga [10]noticed thatfor a given wing-tank area (which determines the restoring term in the natural period) the height of the connecting duct is the remaining design parameter for the tuning effect. Samoilescu*et al*. [3] mentioned that tank stabilizers are virtually independent of the forward speed of the vessel: they generate anti rolling forces by phased flow of appropriate masses of fluid (water or reserve fuel, etc.) in transverse tanks installed at suitable heights and distances from the ship's centre line. Tanizawa*et al*. [7] used a numerical wave tank (NWT) which was applied to the simulation of coupled motions of ship and ART in beam seas. Holden *et al*. [2] presented two novel nonlinear models of U-shaped ART for ships, and their linearization. The models were derived using Lagrangian mechanics.

Water in the reservoirs of an ARTis expected to oscillate with the same period but exactly out of phase to the vessel's oscillation. For the greatest effect, ARTs are often placed where they would induce the highest counter-rolling moment (often, high up on the vessel). The motivating force for the anti-roll tanks is predominantly the action of gravity on the mass of the fluid. Since the fluid can only flow downhill and has inertia, it cannot start to move until the ship has rolled a few degrees. The natural restoring forces limit the maximum roll angle and initiate a roll in the opposite sense. In the meantime, the fluid continues to flow downhill, accumulating on the still low side and provides a moment opposing the ship motion. As the ship returns and passes its upright position, fluid again flows downhill to repeat the process. The transverse acceleration of the fluid generates an inertia force and thereby a moment, about the roll centre, which reduces the gravity moment when the tanks are below the roll centre and increases it when they are above. Maximum moment is experienced at the vertical position of the ship. The kinetic and potential energiesof the fluid in the tanks are originated from the energy generated during rolling. Part of the energy is dissipated by vortex shedding and fluid viscous effects related to skin friction on the walls of the tank. Thus, the phase lag may be increased, within limits, by placing obstructions e. g. orifice plates, grilles, etc. in the fluid flow path to increase the damping.In contrast to the other papers available on this topic, the present paper takes a new theoretical approach to determine the period of oscillation of water in tanks that can be used to counterfeit anti-rolling motion in ships.

2. Theoretical Analysis

Let us consider two identical tanks each of cross-sectional area A. The tanks are connected at their bottom by a horizontal pipe of length L and cross-sectional area a, as shown in (Figure 1). The tanks are filled with water up to a depth of h.

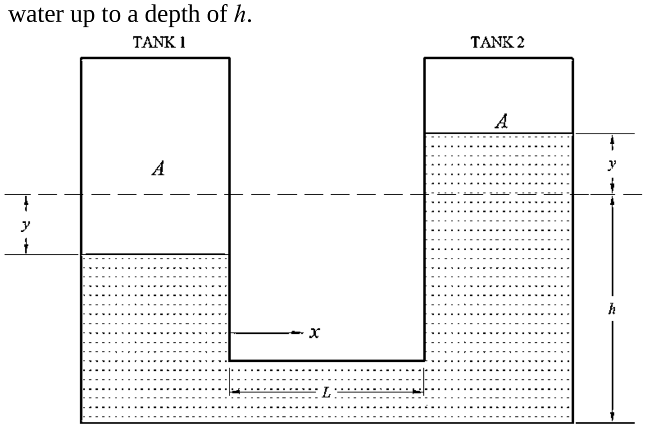

Fig. 1. Natural oscillations of water in two identical tanks connected by a pipe.

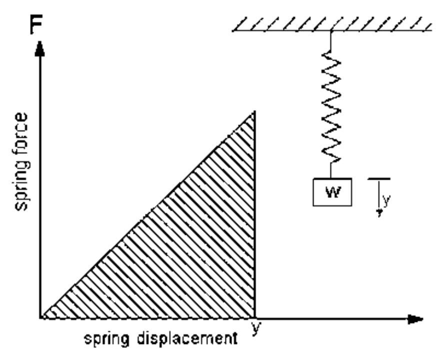

Fig. 2. Simple spring-mass system

Suppose that oscillation occurs in the tanks due to some external disturbance. In such a case, the water level will fall in one tank, with a corresponding rise in the other. The head of water, causing flow in the pipe, will be the difference between the two water levels.Let the water level in Tank 1 fall down by y. Then the water level in Tank 2 will rise up by y. Let x be the horizontal displacement of water in the pipe. The volume of water that has passed from Tank 1 is Ay. From constancy of volume of water, we can write

$$Ay = ax \quad (1)$$

$$\therefore \quad x = \frac{Ay}{a} \quad (2)$$

Using the principle of conservation of energy, the natural frequency of an oscillating system can be easily determined, and in this case, it can be done using an analogy with the spring-mass system. For a vibrating system, the energy is partly kinetic and partly potential. The kinetic energy $K$is associated with the velocity of the mass and the potential energy$P$ is due to the strain energy stored in the spring. Thus,

$$K + P = \text{Total Energy} = \text{constant} \quad (3)$$

Therefore, the rate of change of energy is zero,

$$\frac{d}{dt}(K + P) = 0 \tag{4}$$

The equation of motion of the spring mass system can be derived from energy considerations. The kinetic energy of the mass is given by

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{y}^2 \tag{5}$$

The potential energy of the system is due to strain energy stored in the spring (Figure 2) and is given by

$$P = \int_0^x ky\,dx = \frac{1}{2}\,ky^2 \tag{6}$$

Therefore, one can now easily write down the expressions for the potential energy of the water oscillating in the tanks. In this case, the restoring force is

$$F = 2Ay\rho g \tag{7}$$

$$\therefore \quad k = \frac{2Ay\rho g}{y} = 2A\rho g \tag{8}$$

Combining Eqn (6) and Eqn (8), we get

$$P = A\rho g y^2 \tag{9}$$

$$\therefore \quad \frac{dP}{dt} = 2A\rho g y\dot{y} \tag{10}$$

The kinetic energy of the oscillating column of water is given by

$$K = \frac{1}{2}Ah\rho\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}Ah\rho\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}aL\rho\left(\frac{dx}{dt}\right)^2$$

It can be easily shown from the above that

$$\frac{dT}{dt} = \rho\left[2Ah + aL\left(\frac{A}{a}\right)^2\right]\dot{y}\ddot{y} \tag{11}$$

From Eqn (4),Eqn(10) and Eqn(11), we obtain

$$\rho\left[2Ah + aL\left(\frac{A}{a}\right)^2\right]\dot{y}\ddot{y} + 2A\rho g y\dot{y} = 0$$

Dividing the above expression by $p\dot{y}$ throughout, we get

$$\left[2Ah + aL\left(\frac{A}{a}\right)^2\right]\ddot{y} + 2Agy = 0 \tag{12}$$

or,

$$\ddot{y} + \left[\frac{2Ag}{2Ah + aL(A/a)^2}\right]y = 0$$

which is similar to

$$\ddot{y} + p^2 y = 0 \tag{13}$$

where p is the natural frequency of oscillation in radian/second. Therefore,

$$p^2 = \frac{2Ag}{2Ah + aL(A/a)^2} \tag{14}$$

or,

$$p = \left[\frac{g}{h + \frac{L}{2}\left(\frac{A}{a}\right)}\right]^{1/2} \tag{15}$$

It can be easily shown that for multiple pipes,

$$p = \left[\frac{g}{h + \frac{L}{2}\left(\frac{A}{na}\right)}\right]^{\frac{1}{2}} \tag{16}$$

where *n* is the number of connecting pipes. For rectangular tank connected by rectangular pipe, we get

$$p = \left[\frac{g}{h + \frac{L}{2}\left(\frac{bl}{h'l'}\right)}\right]^{1/2} \tag{17}$$

where *b* and *l*are the breadth and longitudinal extent of the vertical arm of the tanks and h' and $l'$are the height and longitudinal extent of the horizontal pipe respectively.When the longitudinal extent of the rectangular vertical arm and that of the horizontal pipe are same, Eqn (17) reduces to

$$p = \left[\frac{g}{h + \frac{L}{2}\left(\frac{b}{h'}\right)}\right]^{1/2} \quad (18)$$

For two identical square tanks connected by a duct of square cross-section, Eqn (17) gives

$$p = \left[\frac{g}{h + \frac{L}{2}\left(\frac{b}{h'}\right)^2}\right]^{1/2} \quad (19)$$

Similarly for two circular tanks connected by a circular pipe,

$$p = \left[\frac{g}{h + \frac{L}{2}\left(\frac{D}{d}\right)^2}\right]^{1/2} \quad (20)$$

For two identical square tanks connected by a circular pipe,

$$p = \left[\frac{g}{h + \frac{2L}{\pi}\left(\frac{b}{d}\right)^2}\right]^{1/2} \quad (21)$$

The above derivation takes into account an ideal fluid flow condition. Taking the effect of damping into consideration, and D_e as the loss of energy due to damping, Eqn (4) gives

$$\frac{d}{dt}(K + P) = -D_e \quad (22)$$

The energy lost due to damping can be considered as the product of thedamping force and thereduction in height of the water column due to damping. Since the damping force is proportional to the velocity, we get

$$D_e = C\dot{y}h_d \quad (23)$$

Where C is the damping constant and h_d is the reduction in height of the water column due to damping.If $U_d$is the change in potential energy and $T_d$is the change in kinetic energy due to damping,

$$D_e = P_d + K_d \quad (24)$$

Let U_1 andU_2be the potential energy of the water column in ideal fluid and viscous fluid respectively. Therefore, Eqn (9) gives

$$P_1 = A\rho g y^2 \quad (25)$$

therefore,

$$P_2 = A\rho g(y - h_d)^2 \quad (26)$$

Neglecting the second order terms and using the Newton's equation of motion,

$$P_d = P_1 - P_2 = 2A\rho g y h_d \quad (27)$$

$$y = \frac{\dot{y}^2}{2g} \quad (28)$$

$$P_d = A\rho\dot{y}^2 h_d \quad (29)$$

Similarly, the corresponding expressions for the kinetic energy can be written as

$$K_1 = \frac{1}{2}\rho\left[2Ah + aL\left(\frac{A}{a}\right)^2\right]\dot{y}^2 \quad (30)$$

$$K_2 = \frac{1}{2}\rho\left[2A(h - h_d) + aL\left(\frac{A}{a}\right)^2\right]\dot{y}^2 \quad (31)$$

Thus we get $K_D = K_1 - K_2 = A\rho h_d \dot{y}^2$ (32)

Therefore Eqn(24) becomes $D_e = 2A\rho\dot{y}^2 h_d$ (33)

So from Eqn (22),

$$\rho\left[2Ah + aL\left(\frac{A}{a}\right)^2\right]\dot{y}\ddot{y} + 2A\rho g y\dot{y} = -2A\rho\dot{y}^2 h_d \quad (34)$$

which gives

$$\ddot{y} + \left[\frac{2Ag}{2Ah + aL(A/a)^2}\right]y + \left[\frac{2h_d A}{2Ah + aL(A/a)^2}\right]\dot{y} = 0 \quad (35)$$

Let

$$p^2 = \frac{2Ag}{2Ah + aL(A/a)^2} \quad (36)$$

and

$$2n = \frac{2Ah_d}{2Ah + aL(A/a)^2} \quad (37)$$

Then Eqn (35) becomes

$$\ddot{y} + 2n\dot{y} + p^2 y = 0 \quad (38)$$

which is a second order linear homogeneous differential equation. Considering the roots of the characteristic equation real and distinct, and the solution of the equation

$$y = e^{rt} \tag{39}$$

It can be easily shown that

$$r = -n \pm \sqrt{(n^2 - p^2)} \tag{40}$$

So the damped frequency can be expressed as

$$p_d = \sqrt{(p^2 - n^2)} \tag{41}$$

3. Results and Discussion

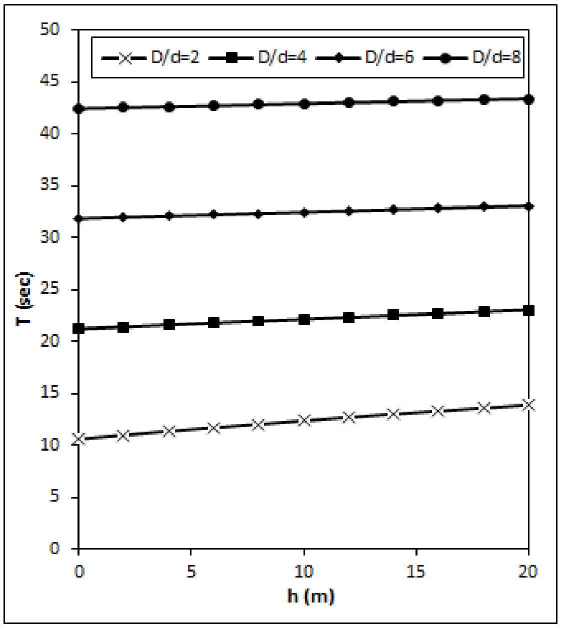

Fig. 3. Variation of the period of oscillation (*T*) with the height (*h*) of the column of water in circular tanks connected by a circular pipe for different values of *D/d*.

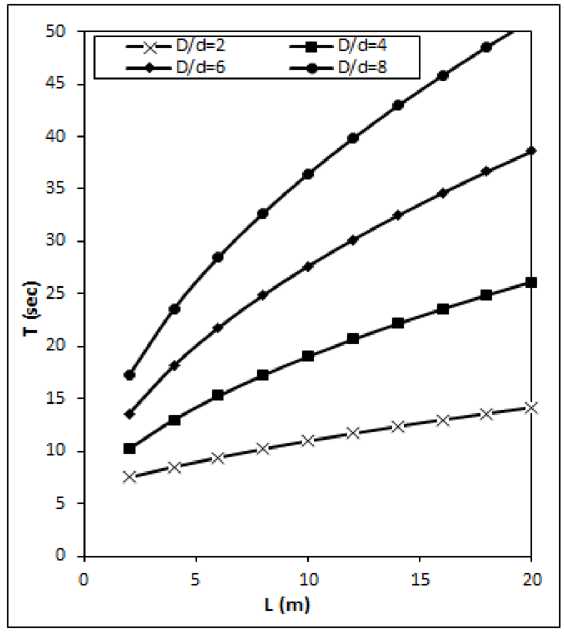

Fig. 4. Variation of the period of oscillation (*T*) with the length (*L*) of the connecting pipe for circular tanks connected by a circular pipe for different values of *D/d*.

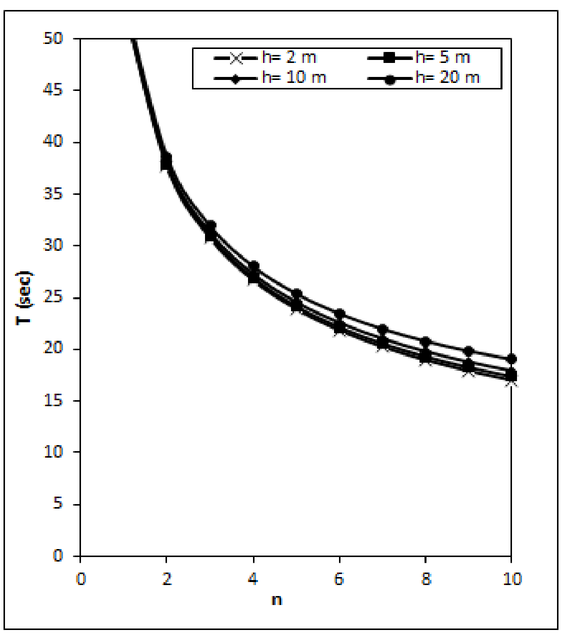

Fig. 5. Variation of the period of oscillation (*T*) with the number of connecting pipes (*n*) for different water heights (*h*) in the upright tanks.

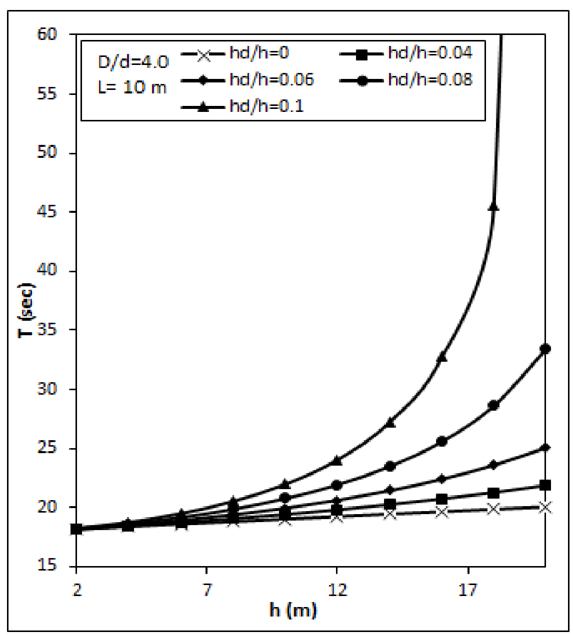

Fig. 6. Variation of the time period (*T*) with the vertical rise of water in upright tanks (*h*) for different damping ratios (*hd/h*).

Based on the theoretical analysis, a number of computer programs have been developed in Fortran 2003 and executed on the Core-2-Quad based processor. The computational results are plotted graphically. It may be noted that the expression for the frequency of oscillation (which has been derived in the preceding section) has been used for finding the period of oscillation (T) using the well-known relationship $T=2\pi/p$. Furthermore, it is to be noted that the period of oscillation has been used as a key parameter for the analysis of results. It is clearly seen from Eqn (15) that the tank parameters (i.e. breadth, diameter and length of the tanks) do not affect the period of oscillation as long as the cross-sectional areas of the tanks are the same. So the results obtained are generalized using only circular tank connected by a circular pipe. The ratios of the diameter of the tank to the connecting pipe have been used to visualize the particular changes associated with the change in areas.Fig. 3shows that as the height of the water column in the vertical tanks increase the time period increases but the rate of increase is very low. It is also observed that the period of oscillation increases rapidly with the increase of the ratio (D/d) where D is the diameter of the vertical tanks and d the diameter of the circular pipe.Fig. 4shows that the period of oscillation increases remarkably as the length of the connecting pipe increases but the variation is not linear. Furthermore, the period of oscillation increases rapidly with the increase of the ratio (D/d).It is seen from Fig. 5 that the period of oscillation can be reduced significantly by increasing the number of connecting pipes. However, no remarkable change in the period of oscillation is observed with the change in the depth of water in the tanks. It is already mentioned in the previous section that additional damping forces arise from the friction between the moving fluid and the surrounding tank wall. This damping effect is clearly observed from Fig. 6that the period of oscillation is found to increase significantly with the increase of the ratio h_d/h where h_d is the loss of height due to damping and h is the depth of water.

4. Conclusions

The following conclusions can be drawn from the present study:

(a) Firstly, an expression for the frequency of oscillation of water in two identical tanks has been derived assuming ideal fluid flow theory. Subsequently, the expression for the frequency of oscillation has been modified taking into consideration the additional damping forces which arise from friction between the moving fluid and the surrounding tank wall. Obviously, this theoretical model is more realistic than the one derived on the assumption of ideal fluid.

(b) Both the circular and the rectangular tanks produce the same period of oscillation provided their cross-sectional areas are equal.

(c) Once the tanks are installed on a ship, the dimensions of the tanks (length, breadth and diameter) cannot be changed. However, the depth of water in the tanks is the only parameter which can be varied to match with the ambient condition.

(d) The period of oscillation is found to decrease sharply as the number of connecting pipes increases.

(e) Finally, the period of oscillation is found to increase with the increase of the depth of water in the tanks. However, the rate of increase is quite low in case of ideal fluid flow assumption. On the other hand, the period of oscillation increases at a rapid rate with the depth of water when the effect of damping is taken into account.

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